A Brief Introduction On Information Causality

References:

[1] Pawłowski M, Paterek T, Kaszlikowski D, et al. Information causality as a physical principle[J]. Nature, 2009, 461(7267): 1101-1104.

[2] Dahlsten O C O, Lercher D, Renner R. Tsirelson's bound from a generalized data processing inequality[J]. New Journal of Physics, 2012, 14(6): 063024.

[3] Allcock J, Brunner N, Pawlowski M, et al. Recovering part of the boundary between quantum and nonquantum correlations from information causality[J]. Physical Review A, 2009, 80(4): 040103.

QM State space: vector space

- Underlying principles?

- In the last decades, it was understood that quantum entanglement plays a crucial role in addressing this question.

- The most direct signature of entanglement are the quantum correlations (QC) obtained by measuring the sub-systems separately.

Quantum correlations

- The set of probability distributions {p(a_1, a_2,…,a_n | x_1, x_2,…,x_n)}, where x_1, x_2,… are inputs for different parties, a_1, a_2,… are their outputs respectively.

Quantum correlations is larger than classical ones

- For CHSH inequality, the boundary for classical correlations is 2, but quantum correlations can reach $2\sqrt{2}$.

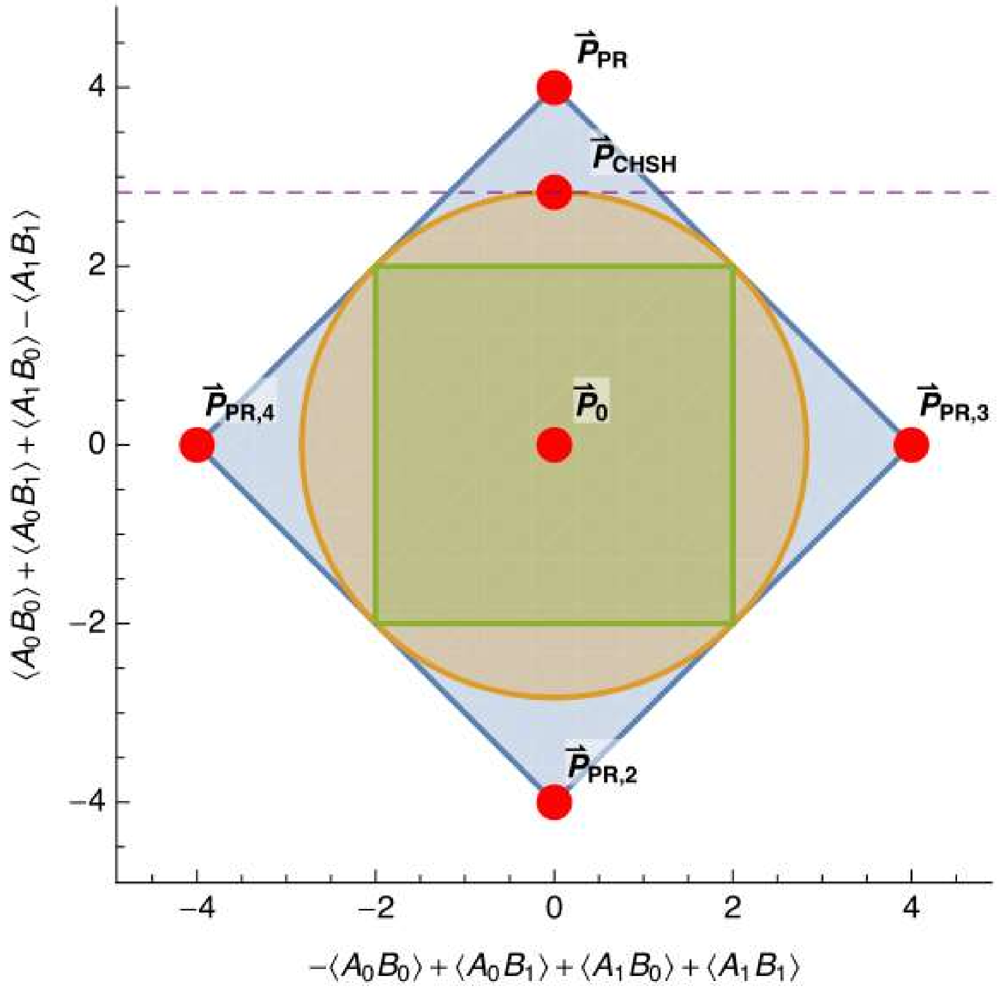
Recovering QC with physical principles

Bounding the allowed set of QC by physical principles, instead of the form of QM.

Non-signaling (NS) principle

$$\sum_{a_j} P\left(a_1, \ldots, a_j, \ldots, a_n | x_1, \ldots, x_j, \ldots, x_n\right) =$$
$$\sum_{a_j} P\left(a_1, \ldots, a_j, \ldots, a_n | x_1, \ldots, x'_j, \ldots, x_n\right)$$
$$\forall j \in [n], \{a_1, \ldots, a_n\} \setminus a_j, \{x_1, \ldots, x_j, x'_j, \ldots, x_n\}$$

- Non-signaling correlations can attain the boundary 4 ($>2\sqrt{2}$) for CHSH inequality.



Stronger principles are needed to recover quantum boundaries.

[1] Goh K T, Kaniewski J, Wolfe E, et al. Geometry of the set of quantum correlations[J]. Physical Review A, 2018, 97(2): 022104.

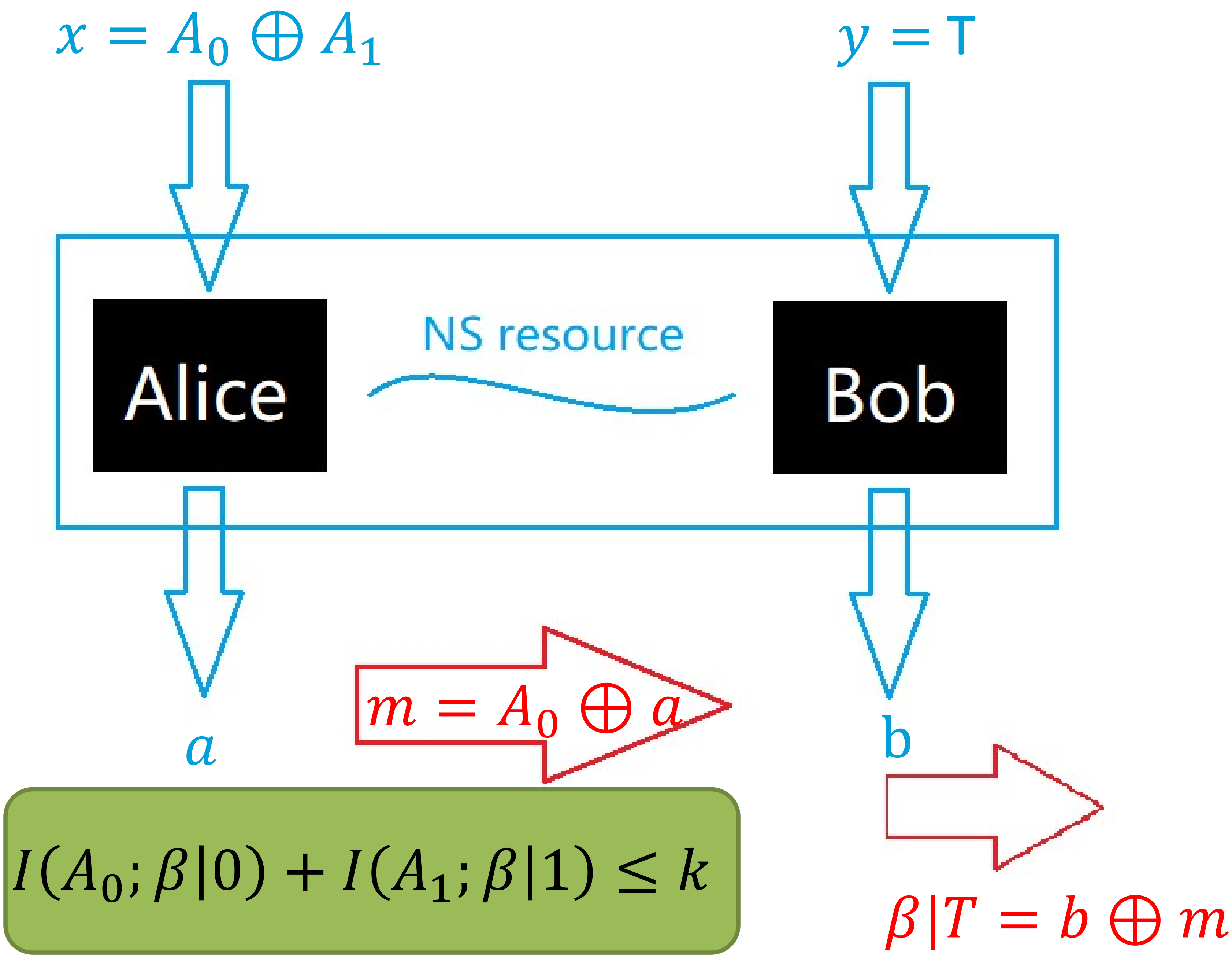
Information Causality

- In a communication scenario assisted with NS resources, the potential information obtainable will not exceed the information of the message sent (channel capacity).

By this principle we can place stronger boundaries for the resources used than NS principle.

[2] Pawłowski M, Paterek T, Kaszlikowski D, et al. Information causality as a physical principle[J]. Nature, 2009, 461(7267): 1101-1104.

Random bits $A_0, A_1 = \{0,1\}, T = \{0,1\}$. One bit of communication is allowed.



- Quantum correlations satisfy IC inequality.
- However, some NS correlations violate it. For example, when the channel is perfect (k=1), if Alice and Bob share a PR-box, $I(A_0;\beta|0)$=1, $I(A_1;\beta|1)$=1

$$I(A_0;\beta|0) + I(A_1;\beta|1) = 2 > k$$

Proof that QM satisfies IC

- $I(A_i;\beta|i) \leq I(A_i;\ \rho_{Bm})$. By data-processing inequality $H(A|B) \leq H(A|B')$.

- $I(A_0;\ \rho_{Bm}) + I(A_1;\ \rho_{Bm}) \leq I(A;\ \rho_{Bm}) \leq$k

On general theory cases

- A general probabilistic model satisfying the following conditions satisfies IC

Definition 5 (DPI). *Consider two systems A and B. The DPI is that for any allowed state $\vec{P}_{AB} \in \mathcal{S}_{AB}$ and for any allowed local transformation $T : \vec{P}_B \to \vec{P}'_B$*

$$H(A|B)_{\vec{P}_{AB}} \leqslant H(A|B')_{(\mathbb{1}\otimes T)\vec{P}_{AB}}. \tag{A.1}$$

Definition 6 (Conditional entropy (COND)). *The conditional entropy $H(A|B)$, however it is defined, must for all allowed states on AB satisfy*

$$H(A|B) = H(AB) - H(B). \tag{A.2}$$

Definition 7 (Reduction to Shannon entropy (SHAN)). *The entropy H must reduce to the Shannon entropy for classical systems.*

[3] Dahlsten O C O, Lercher D, Renner R. Tsirelson's bound from a generalized data processing inequality[J]. New Journal of Physics, 2012, 14(6): 063024.

Possible explanation of the potential information

- The mutual information $I(A;\ B_m)$ of A and the joint system B_m.

- The role of IC protocol is to obtain a value which is close but does not exceed $I(A;\ B_m)$.

Open question

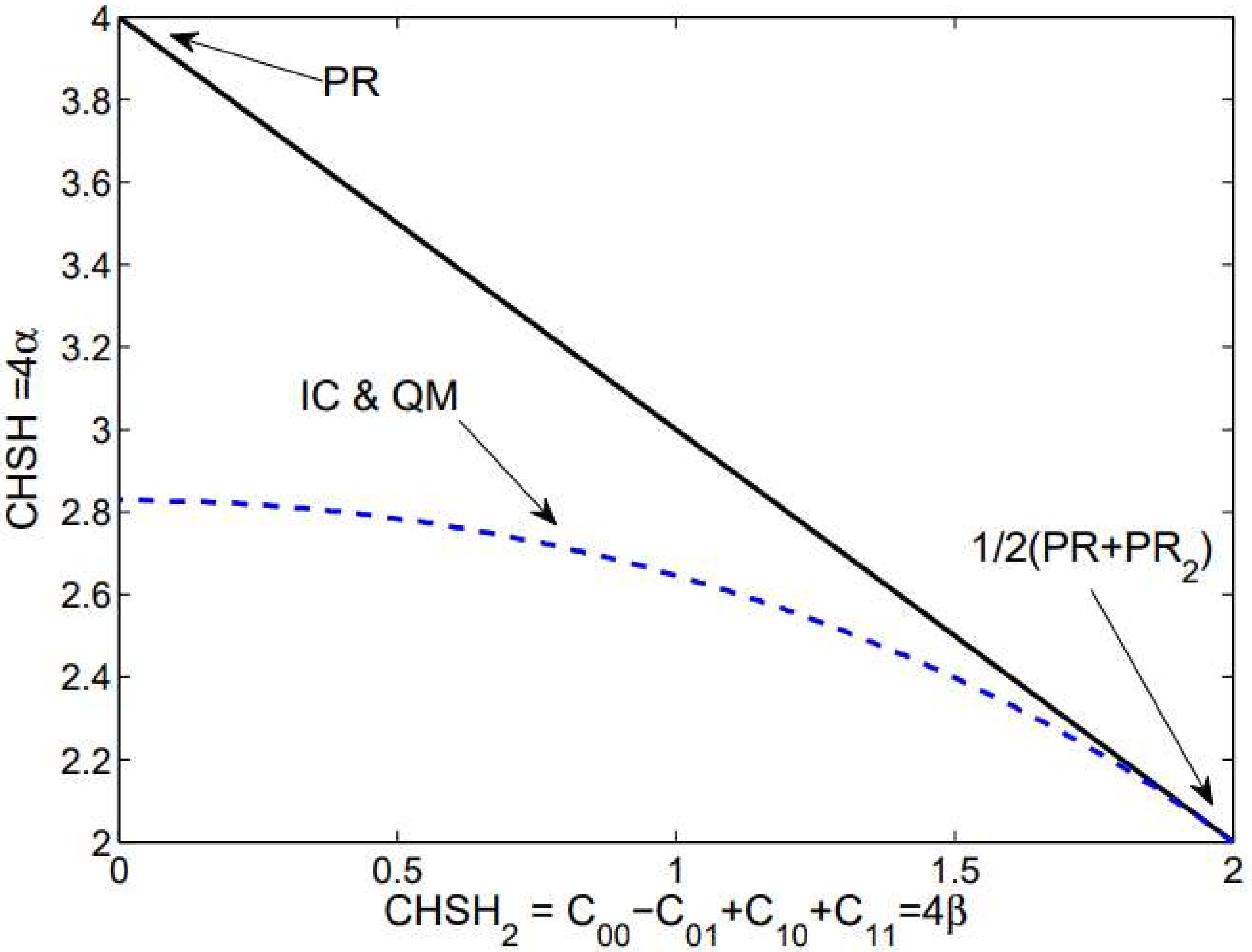
- Does IC principle recover all boundaries of quantum correlations?

- Ruling out “Almost Quantum Correlations (AQC)”.

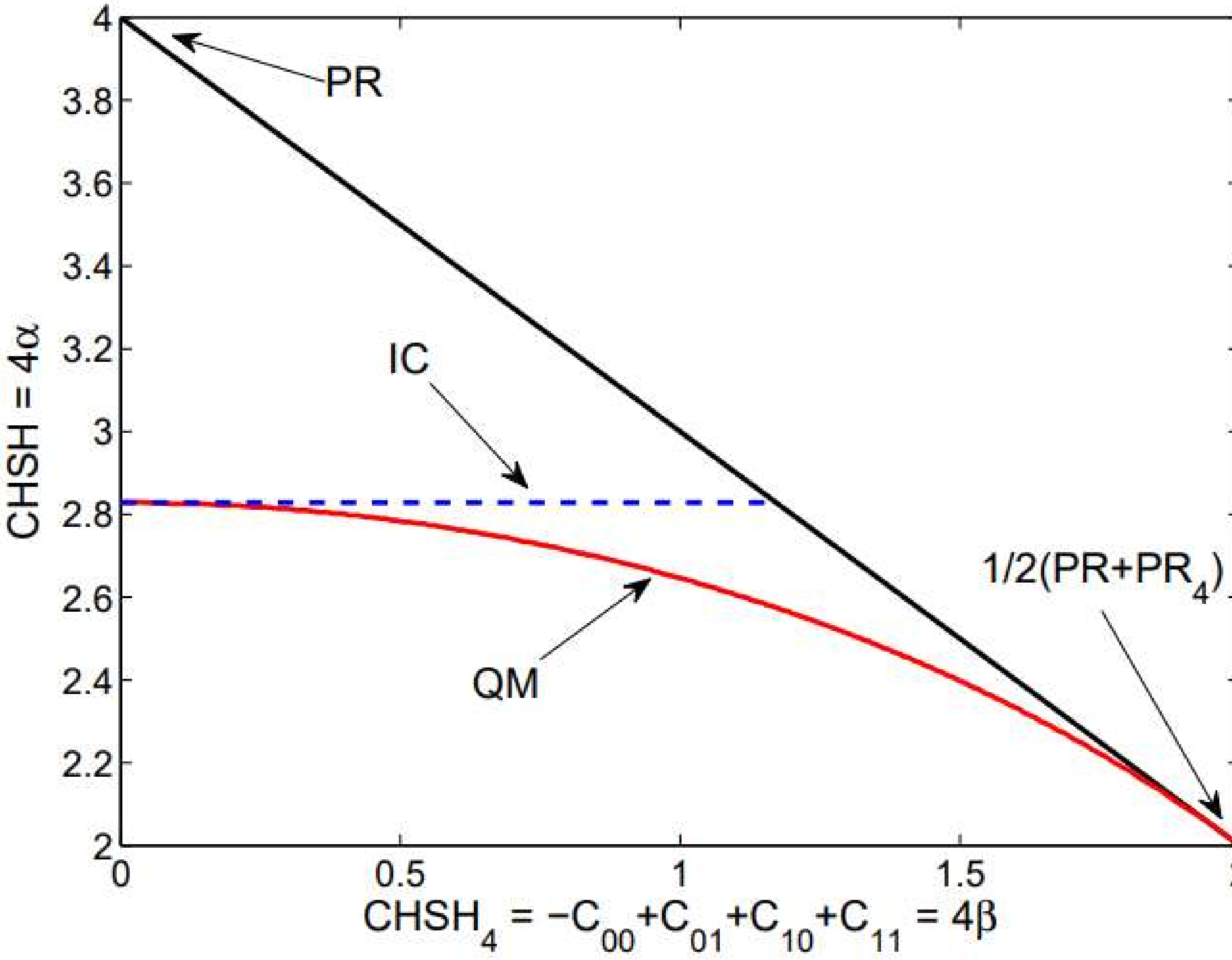
[4]Navascués M, Guryanova Y, Hoban M J, et al. Almost quantum correlations[J]. Nature communications, 2015, 6(1): 1-7.

Reference for the following slices:

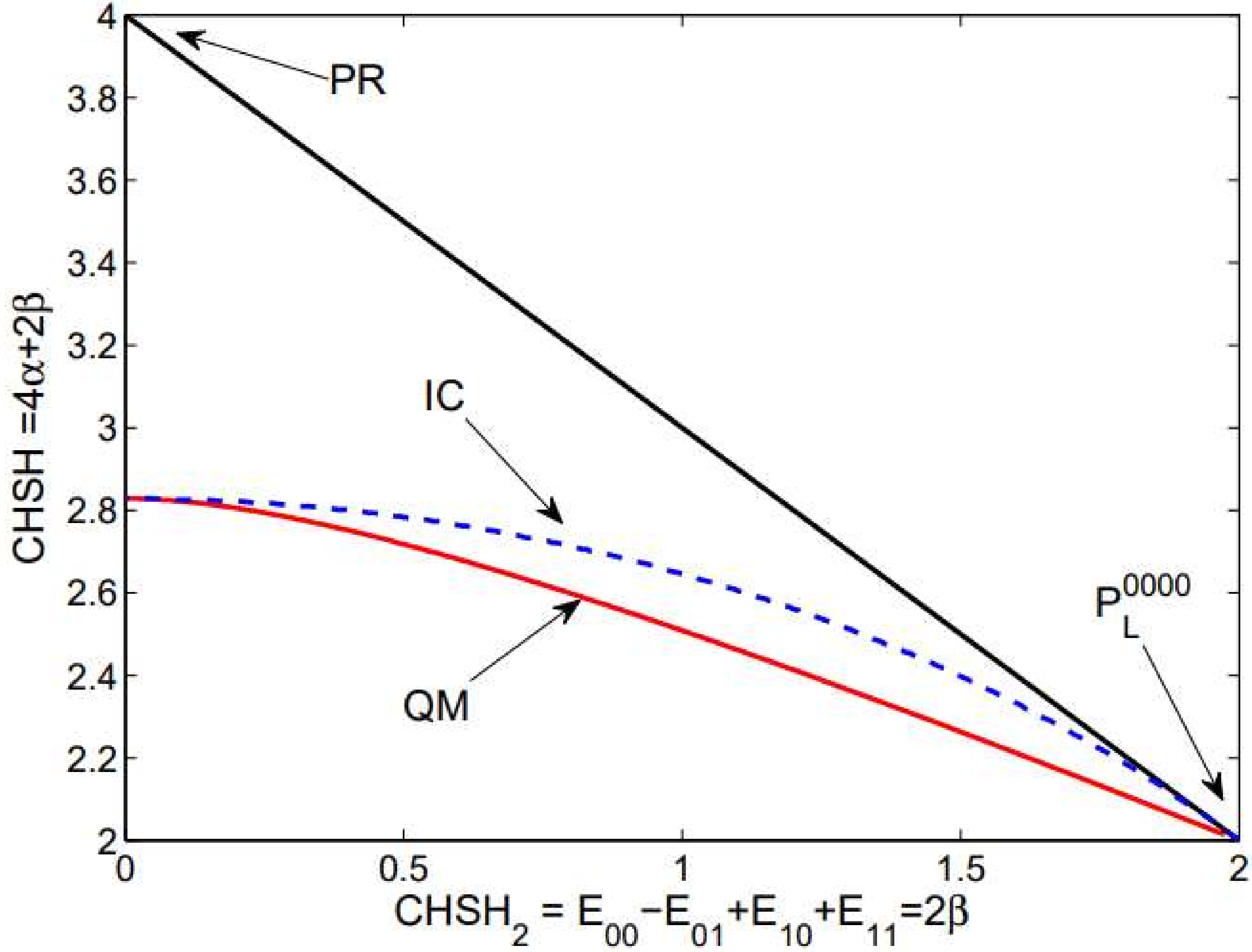
[5] Allcock J, Brunner N, Pawlowski M, et al. Recovering part of the boundary between quantum and nonquantum correlations from information causality[J]. Physical Review A, 2009, 80(4): 040103.



- In this slice, AQC, QC, and IC correlations coincide.



- In this slice, AQC and QC coincide, but IC correlations form a larger set.



- In this slice, AQC set is slightly larger QC set, IC correlations form a larger set.

- We are working to improve the IC boundary in the former slices by modifying IC inequality.

- Thank you!