

## §.2. 様々な行列 $tA$

### §.2.1 転置と共役

#### Def (転置行列)

$(m \times n)$  行列  $A = (a_{ij})_{i,j}$  に対し,  $A$  の行と列を  $\lambda$  だけ換えて得られる  $(n \times m)$  行列  $(a_{ji})_{j,i} \in A$  の転置行列といひ  ${}^tA$  とかく.  $\square$

#### Eq. (例)

$$A = \begin{pmatrix} \boxed{1} & \boxed{2} & \boxed{3} \\ \boxed{4} & \boxed{5} & \boxed{6} \end{pmatrix} \rightsquigarrow {}^tA = \begin{pmatrix} \boxed{1} & \boxed{4} \\ \boxed{2} & \boxed{5} \\ \boxed{3} & \boxed{6} \end{pmatrix}$$

$$B = \begin{pmatrix} \boxed{1} & \boxed{2} \\ \boxed{3} & \boxed{4} \end{pmatrix} \rightsquigarrow {}^tB = \begin{pmatrix} \boxed{1} & \boxed{3} \\ \boxed{2} & \boxed{4} \end{pmatrix}$$

$$C = (\boxed{1} \ \boxed{2} \ \boxed{3}) \rightsquigarrow {}^tC = \begin{pmatrix} \boxed{1} \\ \boxed{2} \\ \boxed{3} \end{pmatrix} \quad \square$$

Prop (命題)  $A, B$  : 行列.

$$\bullet {}^t({}^tA) = A$$

$$\bullet {}^t(A+B) = {}^tA + {}^tB$$

$$\bullet {}^t(AB) = {}^tB {}^tA \quad (\text{証明略}) \quad \square$$

## Def. (共役行列)

複素行列  $A = (a_{ij})_{ij}$  の各成分  $a_{ij} \in \mathbb{C}$  の複素共役  $\bar{a}_{ij}$  に置き換えてできる行列  $(\bar{a}_{ij})_{ij} \in A$  の共役行列と叫び  $\bar{A}$  とかく。  $\square$

Eg.  $A = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \rightsquigarrow \bar{A} = \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$

$$B = \begin{pmatrix} 1+i & 2 & 0 \\ -1 & 2i & 1-i \end{pmatrix} \rightsquigarrow \bar{B} = \begin{pmatrix} 1-i & 2 & 0 \\ -1 & -2i & 1+i \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 \\ 0 & i \end{pmatrix} \rightsquigarrow \bar{C} = \begin{pmatrix} 1 & 2 \\ 0 & i \end{pmatrix} \quad \square$$

Prop  $A, B$  : 行列.

$$\bullet \overline{(\bar{A})} = A$$

$$\bullet \overline{(A+B)} = \bar{A} + \bar{B}$$

$$\bullet \overline{(kA)} = \bar{k} \bar{A} \quad (k \in \mathbb{C})$$

$$\bullet \overline{AB} = \bar{A} \bar{B} \quad (\text{証明略}) \quad \square$$

Def (随伴行列 または エルミート共役行列)

行列  $A = (a_{ij})_{ij}$  に対し

$$A^* := \overline{A^T} = (\overline{a_{ji}})_{ji}$$

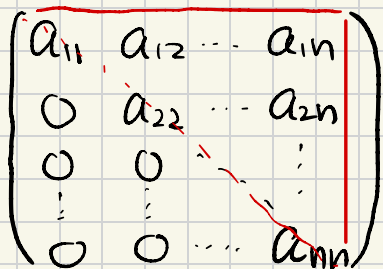
$\exists A$  の随伴行列  $A^*$  は エルミート共役行列といふ  $\square$

(例)

Eg  $A = \begin{pmatrix} 2+i & 1 & i \\ -1 & 1 & 2+i \end{pmatrix} \rightsquigarrow A^* = \begin{pmatrix} 2-i & 1 \\ 1 & 1 \\ -i & 2-i \end{pmatrix}$

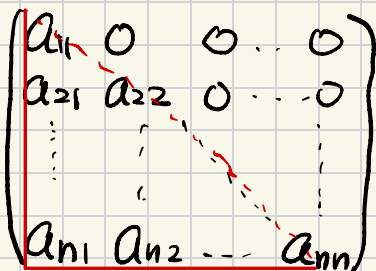
$$B = \begin{pmatrix} -i & 2i \\ 3i & i \end{pmatrix} \rightsquigarrow B^* = \begin{pmatrix} i & -3i \\ -2i & -i \end{pmatrix} \square$$

### §.2.2 特殊な正方行列



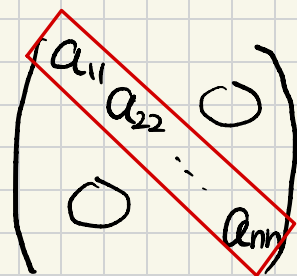
A diagram of an upper triangular matrix. The matrix is enclosed in large parentheses. The diagonal elements  $a_{11}, a_{22}, \dots, a_{nn}$  are marked with red dots. A red dashed line runs from the top-left to the bottom-right. A red vertical line is drawn to the right of the matrix, and a red horizontal line is drawn below the matrix, forming a red L-shape that highlights the upper triangular region.

上三角行列



A diagram of a lower triangular matrix. The matrix is enclosed in large parentheses. The diagonal elements  $a_{11}, a_{22}, \dots, a_{nn}$  are marked with red dots. A red dashed line runs from the top-left to the bottom-right. A red vertical line is drawn to the left of the matrix, and a red horizontal line is drawn below the matrix, forming a red L-shape that highlights the lower triangular region.

下三角行列



A diagram of a diagonal matrix. The matrix is enclosed in large parentheses. The diagonal elements  $a_{11}, a_{22}, \dots, a_{nn}$  are marked with red dots. A red dashed line runs from the top-left to the bottom-right. A red diamond shape is drawn around the diagonal elements, highlighting the diagonal.

対角行列

!!

$\text{diag}(a_{11}, a_{22}, \dots, a_{nn})$

Eg.  $\begin{pmatrix} 2 & 4 \\ 0 & 3 \end{pmatrix}$ ,  $\begin{pmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 1 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$

上三角                      下三角                      对角

⊙ スカラー-行列:  $\text{diag}(a, a, \dots, a) = \begin{pmatrix} a & & 0 \\ & \ddots & \\ 0 & & a \end{pmatrix}$

Eg.  $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2c & 2d \end{pmatrix} = 2 \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

スカラー-倍!

⊙ 単位行列:  $\text{diag}(\underbrace{1, 1, \dots, 1}_n) = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = E_n$

Rmk.  $E_n A = A E_n = A$  ( $A: (n \times n)$  行列)

⊙ 特殊な実行列

$A = (a_{ij})_{1 \leq i, j \leq n}$  ( $a_{ij} \in \mathbb{R}$ )

条件	名称
$A^t A = {}^t A A = E_n$	A: 直交行列
${}^t A = A$	A: 対称行列
${}^t A = -A$	A: 交代行列

□

Eg  $A = \begin{pmatrix} 2 & -1 \\ -1 & 5 \end{pmatrix}$ ,  $B = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

对称
直交
交代

⑩ 特殊な複素行列

$$A = (a_{ij})_{1 \leq i, j \leq n} \quad (a_{ij} \in \mathbb{C})$$

条件	名称
$AA^* = A^*A = E$	A: $\mathbb{C}$ -ユニタリ行列
$A^* = A$	A: $\mathbb{C}$ -エルミート行列
$A^* = -A$	A: 反エルミート行列 <span style="float: right;">□</span>

Eg.

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}, \quad B = \begin{pmatrix} & \\ & \end{pmatrix}, \quad C = \begin{pmatrix} & \\ & \end{pmatrix}$$

$\mathbb{C}$ -ユニタリ

↑
↗

例 2.20, 2.27, 5.5 HW