ABSTRACTS

Neal Bez (*University of Glasgow*) . . . . Sep. 29th (Tue), 14:30–15:20

Nonlinear Brascamp–Lieb inequalities via induction-on-scales

The Brascamp–Lieb inequalities simultaneously generalise important classical inequalities such as the multilinear Hölder, sharp Young convolution and Loomis–Whitney inequalities. We will use the method of induction-on-scales to prove certain diffeomorphism invariant nonlinear Brascamp–Lieb inequalities and outline applications of such inequalities to problems in harmonic analysis. This is joint work with Jon Bennett.

Dong Hyun Cho (*Kyonggi University, Korea*) . . . . Oct. 1st (Thu), 10:00–10:50

An analogue of operator-valued Feynman integral via conditional Feynman integrals on a function space

In this talk, we introduce a simple formula for the conditional expectations over an analogue $C[0,t]$ of the Wiener space, the space of continuous real-valued paths on the interval $[0,t]$. Using this formula, we evaluate the analogue of the conditional analytic Feynman integral for the functional

$$F(x) = \exp\left\{ \int_0^t \theta(s,x(s))\, d\eta(s) \right\} \phi(x(t))$$

which is defined on $C[0,t]$ and is of interest in Feynman integration theories and quantum mechanics. We introduce an integral transform as an operator-valued Feynman integral over $C[0,t]$ and evaluate the integral transform for the function F via the conditional Feynman integral over generalized Wiener paths as a kernel.

Xuan Duong (*Macquarie University*) . . . . Sep. 28th (Mon), 15:40–16:30

Hardy spaces associated to operators satisfying Davies-Gaffney estimates and bounded H_∞ functional calculus

In this talk, we shall give a brief summary of some recent progress of the theory of Hardy spaces, then we develop a theory of Hardy spaces associated with operators and give applications on boundedness of singular integrals. The framework of our theory is as follows.

Let X be a space of homogeneous type. Assume that an operator L has a bounded holomorphic functional calculus on $L^2(X)$ and the kernel of the heat semigroup $\{e^{-tL}\}_{t>0}$ satisfies the Davies-Gaffney estimates. We develop a theory of Hardy spaces $H^p_L(X)$, $0 < p \leq 1$, which includes a molecular decomposition, a square function characterization, duality of Hardy and Lipschitz spaces, and an interpolation theorem. We also show that L has a bounded holomorphic functional calculus on $H^p_L(X)$ for all $p > 0$ and certain Riesz transforms associated with L are bounded from $H^p_L(X)$ to $L^p(X)$ for all $0 < p \leq 2$.

This is joint work with Ji Li (Zhongshan University, Guangzhou, China).

Karlheinz Gröchenig (*University of Vienna*) . . . . Sep. 28th (Mon), 13:30–14:20

Time-frequency methods for pseudodifferential calculus

Time-frequency analysis (also called phase-space analysis) is a local version of Fourier analysis and offers effective tools for the analysis of pseudodifferential operators, mainly for pseudodifferential operators based on a constant geometry. This talk offers a survey of the time-frequency approach to pseudodifferential. We will discuss the appropriate function spaces (modulation spaces) and symbol classes (the Sjöstrand class) and emphasize the functional calculus of pseudodifferential operators, in particular the inversion and spectral invariance of pseudodifferential operators in the Sjöstrand class.

Alexandru Ionescu (*University of Wisconsin-Madison*) . Sep. 29th (Tue), 11:00–11:50

On the regularity of certain spin models in $2+1$ dimensions

I will discuss some recent work on the well-posedness and the stability of certain spin models in two dimensions: the Heisenberg model, the hyperbolic-elliptic Ishimori system, and the incompressible spin fluid system. This is joint work with I. Bejenaru, C. E. Kenig, and D. Tataru.

Masaharu Kobayashi (*Tokyo University of Science*) . . . Sep. 30th (Wed), 15:40–16:30

A Schatten p-class property of pseudo-differential operators with symbols in modulation spaces

In this talk, we discuss sufficient conditions on symbols to ensure the corresponding pseudo-differential operators to belong to the Schatten p-class C_p. For the symbol classes, we use the modulation spaces $M^{p,p}$ and the weighted modulation spaces $M^{2,2}_m$.

This is joint work with Akihiko Miyachi (Tokyo Woman's Christian University).

Hideo Kozono (*Tohoku University*) . . . . Sep. 29th (Tue), 15:40–16:30

Leray's inequality in 3D domains

This is the joint work with Prof. Taku YANAGISAWA at Nara Women's University.

Consider the stationary Navier-Stokes equations in a 3D bounded domain Ω whose boundary $\partial\Omega$ consists of $L+1$ disjoint closed surfaces $\Gamma_0, \Gamma_1, \cdots, \Gamma_L$ with $\Gamma_1, \cdots, \Gamma_L$ inside of Γ_0. The Leray inequality of the given boundary data β on $\partial\Omega$ plays an important role for the existence of solutions. It is known that if the flux $\gamma_i \equiv \int_{\Gamma_i} \beta \cdot \nu dS = 0$ on Γ_i(ν: the unit outer normal to Γ_i) is zero for each $i = 0, 1, \cdots, L$, then the Leray inequality holds. We prove that if there exists a sphere S in Ω separating $\partial\Omega$ in such a way that $\Gamma_1, \cdots, \Gamma_k$, $1 \leqq k \leqq L$ are contained in S and that $\Gamma_{k+1}, \cdots, \Gamma_L$ are in the outside of S, then the Leray inequality necessarily implies that $\gamma_1 + \cdots + \gamma_k = 0$. In particular, suppose that for each each $i = 1, \cdots, L$ there exists a sphere S_i in Ω such that S_i contains only one Γ_i. Then the Leray inequality holds if and only if $\gamma_0 = \gamma_1 = \cdots = \gamma_L = 0$.

Camil Muscalu (*Cornell University*) . . . . Sep. 29th (Tue), 13:30–14:20

Flag paraproducts and non-linear PDE

We will describe the theory of a new class of multi-linear operators which we named ”flag paraproducts” and their relationship with non-linear PDE.

Kenji Nakanishi (*Kyoto University*) . . . . Sep. 30th (Wed), 13:30–14:20

Modified wave operators in L^2 for the Hartree equation

We study asymptotic behavior for large time of solutions for the Hartree equation with the usual nonlinearity of the Coulomb potential, either focusing or defocusing. It is well known that there are global solutions with dispersive behavior for large time, which differs from the free Schrödinger equation (the linearized equation) by some phase correction, just as in the linear scattering theory for the "long-range" potentials.

Construction of such nonlinear global solutions for given asymptotic profiles, or of "nonlinear modified wave operators", goes back to the work of T. Ozawa for the cubic nonlinear Schrödinger equation in one spatial dimension, and it has been extended to the Hartree and other equations, including higher dimensions, and for large asymptotic data. However it is not well understood how general such global behavior is in any case, even for defocusing nonlinearities.

In this talk I will prove that for the Hartree equation in dimensions higher than 4, there exists a global solutions for any given L^2 data, with the corresponding asymptotic profile in the strong topology of L^2. This problem belongs to the so-called "supercritical case" from the scaling view point. That is, one can observe that the nonlinear term is stronger than the linear dispersive part by a simple scaling argument. Therefore we cannot use any perturbative argument by linear estimates, instead the proof goes by a compactness argument, in a similar way as for the "short-range" case in my previous work.

The main difficulty of this argument consists in showing uniform weak continuity of the solutions at time infinity, just by using the L^2 conservation law. The special difficulty of the long rage case is that additional decay has to be derived from the phase correction term, but it is usually carried out in perturbative ways assuming better properties of the solution coming from rapid decay of the asymptotic profile, which is not available in our case. It is overcome by using the additional regularity of the Coulomb potential in higher dimensions in a commutator estimate.

Takayoshi Ogawa (*Tohoku University*) .... Sep. 29th (Tue), 16:40–17:30

Drift-diffusion system in critical cases

We consider the following Cauchy problem for the drift-diffusion system:

$$\begin{cases} \partial_t\rho - \Delta\rho^\alpha + \kappa\nabla\cdot(\rho\nabla\psi) = 0, & x\in\mathbb{R}^n,\ t>0, \\ -\Delta\psi = \rho, & x\in\mathbb{R}^n,\ t>0, \\ \rho(0,x) = \rho_0(x) > 0, & x\in\mathbb{R}^n. \end{cases} \tag{1}$$

where ρ denotes the density function and ψ is unknown potential determined by the density, $\kappa = \pm 1$ and $\alpha \geq 1$ is the adiabatic constant. The sign of the constant κ gives two different situations. Namely when $\kappa = -1$ the system has a large data global solution while when $\kappa = 1$, then the solution may blows up in a finite time. Blanchet-Dobeault-Perthame give the existence of global solution and discussed the asymptotic behavior using the entropy functional when $\alpha = 1$, $n = 2$ and $\kappa = 1$.

We show the following existence results. [1]

(I) Let $\alpha = 1$, $\kappa = \pm 1$ and $n = 2$. For $\rho_0 \geq 0$, $\rho_0 \in L^1(\mathbb{R}^2)$ with $\log(1+|x|)\rho_0(x) \in L^1(\mathbb{R}^2)$, there exists a unique local solution (ρ,ψ) of (1) with $\rho \in C([0,T);L^1(\mathbb{R}^2)) \cap C((0,T);L^{4/3}(\mathbb{R}^2))$, the solution depends on the initial data continuously.

(II) Let $\alpha = 1$, $n = 2$, $\kappa = 1$ and data is nonnegative. Under the condition of (1) with

$$\|\rho(t)\|_1 < 8\pi,$$

then the positive solution is globally exists.

[1]Presented results are a series of joint works with Toshitaka Nagai, Senjo Shimizu and Masaki Kurokiba.

(III) Let $\alpha = 1$, $n = 2$ and $\kappa = 1$ and $\rho_0 \geq 0$ with

$$\|\rho(t)\|_1 > 8\pi$$

then the positive solution blows up in a finite time.

(IV) Let $\alpha = 1$, $n = 2$ and $\kappa = \pm 1$. Assume that the data is in $\mathcal{H}^1(\mathbb{R}^2)$ (Hardy space). Then there exists a local solution $\rho \in C([0,T);\mathcal{H}^1)$ with $\nabla\rho \in L^2(0,T;\mathcal{H}^1)$. The solution obtained in this case is a sign changing solution.

For the critical case of higher dimensional case, we also know the threshold value of the global existence and the finite time blow up. Let

$$H(\rho) = \frac{1}{\alpha - 1}\|\rho\|_\alpha^\alpha - \frac{1}{2}\int_{\mathbb{R}^n} \rho\psi dx.$$

Let $\alpha = \frac{2n}{n+2}$, $n \geq 3$, $\rho_0 \geq 0$, $\kappa = 1$. For $\rho_0 \geq 0$ and $\rho_0 \in L^{\frac{2n}{n+2}} \cap L^1_2(\mathbb{R}^n)$, we assume that $H(\rho_0) < H(U)$.

(V) Then if

$$\|\rho_0\|_{\frac{2n}{n+2}} < \|U\|_{\frac{2n}{n+2}}$$

then a weak solution exists globally in time

(VI) If

$$\|\rho_0\|_{\frac{2n}{n+2}} > \|U\|_{\frac{2n}{n+2}}$$

then the weak solution blows up in finite time, where U be the unique radial solution of

$$-\frac{2n}{n-2}\Delta U^{\frac{n-2}{n+2}} = U, \qquad x \in \mathbb{R}^n,$$

which gives the best possible constant of the Sobolev inequality in $\mathbb{R}^n$.

Stevan Pilipović (*University of Novi Sad*) . . . . Sep. 30th (Wed), 16:40–17:30
Wave front sets in Fourier Lebesgue spaces

We study the wave front set $WF_{\mathcal{F}L^q_{(\omega)}}$ within weighted Fourier Lebesgue spaces $\mathcal{F}L^q_{(\omega)}$, where ω is an appropriate weight function and $q \in [1,\infty]$. We introduce a discrete version of the wave front set and prove that the wave front set of an f can be described by the coefficients of the Gabor expansion of f by the use of admisible Gabor pairs. We prove that

$$WF_{\mathcal{F}L^q_{(\omega_2)}}(a(x,D)f) \subseteq WF_{\mathcal{F}L^q_{(\omega_1)}}(f) \subseteq WF_{\mathcal{F}L^q_{(\omega_2)}}(a(x,D)f) \bigcup \mathrm{Char}(a)$$

for appropriate ω_1, ω_2. Here $\mathrm{Char}(a)$ is the characteristic set of a pseudo-differential operators a.

Also, by the analysis of the wave front set for products we give a hypoellipticity result for a semilinear equation

$$P(x,D)u = F(x, J_k u)$$

where P is a differential polynomial of order m with smooth coefficients and F is a polynomial of order $\tilde{m}$ with elements of the $k-$th jet $J_k u$, $k < m$ and constant coefficients or coefficients in $\mathcal{F}L^1_{(\omega)}$.

This is a joint work with Nenad Teofanov of Novi Sad University and Joachim Toft and his student Karoline Johansson of Växjö University.

Luigi Rodino (*University of Turin*) .... Sep. 30th (Wed), 11:00–11:50

Global regularity for partial differential operators with polynomial coefficients

We consider linear partial differential operators P in $\mathbb{R}^n$. We say that P is globally regular if u belongs to the Schwartz space S , whenever Pu belongs to S and u is a Schwartz distribution. We give some sufficient conditions for the global regularity of P , by using the general Hormander pseudodifferential calculus. Our results extend the results for the globally elliptic operators of Shubin and their nonhomogeneous extension of Boggiatto, Buzano, Rodino, and also include the SG elliptic operators of Cordes.

Michael Ruzhansky (*Imperial College London*) .... Oct. 1st (Thu), 11:00–11:50

Pseudo-differential operators and symmetries

We present an approach to pseudo-differential operators globally on compact Lie groups, without resorting to local charts. We obtain a full global symbol and global calculus. This can be done by presenting functions on the group by Fourier series obtained from the representations of the group. This yields a quanisation of pseudo-differential operators in terms of the matrix-valued symbols with dimensions of matrices in the symbol equal to the dimensions of the representations of the group.

This work is joint with V. Turunen (Helsinki University of Technology). The complete treatise can be found in the following monograph: M. Ruzhansky, V. Turunen: Pseudo-Differential Operators and Symmetries. Birkhauser 2009.

Shuichi Sato (*Kanazawa University*) .... Sep. 28th (Mon), 16:40–17:30

Estimates for singular integrals associated with nonisotropic dilations

Let P be an $n \times n$ real matrix whose eigenvalues have positive real parts, where $n \geq 2$. Define a dilation family $\{A_t\}_{t>0}$ on $\mathbb{R}^n$ by $A_t = t^P = \exp((\log t)P)$.

We prove weak type $(1,1)$ estimates for rough singular integrals on $\mathbb{R}^2$ under the $L \log L$ condition on their kernels that are homogeneous of degree $-\gamma$ with respect to the dilations A_t, where $\gamma = \text{trace}\, P$. Also, we prove some L^p and weighted L^p estimates for certain singular integrals and maximal singular integrals on homogeneous groups, where relevant dilations are defined by choosing P to be a diagonal matrix.

Hideo Takaoka (*Hokkaido University*) .... Sep. 29th (Tue), 10:00–10:50

Growth of Sobolev norms of solutions for the cubic NLS on $\mathbb{T}^2$

In this talk, we consider the cubic nonlinear Schrödinger equation on the two-dimensional torus. If the Hamiltonian yields an apriori bound on the H^1-norm of solutions, the investigation of a byproduct of the local well-posedness permits us to establish the bound of H^1-norm and the order of growth in time of higher Sobolev norms of solutions. The problem we are concerned with is examples of smooth data for which solutions increases with growth in $H^s,\ s > 1$. This is a joint work with J. Colliander, M. Keel, G. Staffilani. T. Tao.

Naohito Tomita (*Osaka University*) .... Sep. 30th (Wed), 10:00–10:50

On the L^p-boundedness of pseudo-differential operators with non-regular symbols

Sjöstrand [2] proved the L^2-boundedness of pseudo-differential operators with symbols in the modulation space $M^{\infty,1}(\mathbb{R}^n \times \mathbb{R}^n)$, where $M^{\infty,1}$ is defined as follows: Let $\varphi \in \mathcal{S}(\mathbb{R}^n)$ be such that $\operatorname{supp}\varphi$ is compact and

$$\sum_{k\in\mathbb{Z}^n} \varphi(\xi - k) = 1 \qquad \text{for all } \xi \in \mathbb{R}^n.$$

Then the modulation space $M^{\infty,1}(\mathbb{R}^n \times \mathbb{R}^n)$ consists of all $\sigma \in \mathcal{S}'(\mathbb{R}^n \times \mathbb{R}^n)$ such that

$$\|\sigma\|_{M^{\infty,1}} = \sum_{k\in\mathbb{Z}^n}\sum_{\ell\in\mathbb{Z}^n} \|\mathcal{F}^{-1}_{(y,\eta)\to(x,\xi)}[\varphi(y-k)\,\varphi(\eta-\ell)\,\widehat{\sigma}(y,\eta)]\|_{L^\infty_{x,\xi}} < \infty.$$

On the other hand, Sugimoto [3] showed the L^2-boundedness for symbols in the Besov space $B^{\infty,1}_{(n/2,n/2)}(\mathbb{R}^n \times \mathbb{R}^n)$, where $B^{\infty,1}_{(n/2,n/2)}$ is defined as follows: Let $\psi_0, \psi \in \mathcal{S}(\mathbb{R}^n)$ be such that

$$\operatorname{supp}\psi_0 \subset \{\xi \in \mathbb{R}^n : |\xi| \le 2\}, \qquad \operatorname{supp}\psi \subset \{\xi \in \mathbb{R}^n : 2^{-1} \le |\xi| \le 2\},$$
$$\psi_0(\xi) + \sum_{j=1}^{\infty} \psi(2^{-j}\xi) = 1 \quad \text{for all } \xi \in \mathbb{R}^n,$$

and set $\psi_j(\xi) = \psi(2^{-j}\xi)$ if $j \ge 1$. Then the Besov space $B^{\infty,1}_{(n/2,n/2)}(\mathbb{R}^n \times \mathbb{R}^n)$ consists of all $\sigma \in \mathcal{S}'(\mathbb{R}^n \times \mathbb{R}^n)$ such that

$$\|\sigma\|_{B^{\infty,1}_{(n/2,n/2)}} = \sum_{k\in\mathbb{Z}}\sum_{\ell\in\mathbb{Z}} 2^{(k+\ell)n/2} \|\mathcal{F}^{-1}_{(y,\eta)\to(x,\xi)}[\psi_k(y)\,\psi_\ell(\eta)\,\widehat{\sigma}(y,\eta)]\|_{L^\infty_{x,\xi}} < \infty.$$

These two results are independent extensions of Calderón-Vaillancourt theorem. However, Boulkhemair [1] pointed out that these independent results can be obtained from the same estimate:

Estimate A There exists a constant $C > 0$ such that

$$\|\sigma(X,D)\|_{\mathcal{L}(L^2)} \le C\,(R_1 \times \cdots \times R_{2n})^{1/2}\,\|\sigma\|_{L^\infty}$$

for all $\sigma(x,\xi) \in L^\infty(\mathbb{R}^n \times \mathbb{R}^n)$ with

$$\operatorname{supp}\widehat{\sigma} \subset \Pi_{i=1}^{2n}[-R_i, R_i],\ R_i \ge 1\ (i = 1, \dots, 2n),$$

where $C > 0$ is independent of $R_i \ge 1$.

Our purpose of this talk is to consider the L^p-boundedness of Estimate A type. As applications, we obtain the L^p-boundedness for symbols in Besov spaces and in modulation spaces.

References

[1] A. Boulkhemair, L^2 estimates for pseudodifferential operators, Ann. Scuola Norm. Sup. Pisa Cl. Sci. 22 (1995), 155-183.

[2] J. Sjöstrand, An algebra of pseudodifferential operators, Math. Res. Lett. 1 (1994), 185-192.

[3] M. Sugimoto, L^p-boundedness of pseudo-differential operators satisfying Besov estimates I, J. Math. Soc. Japan 40 (1988), 105-122.

Baoxiang Wang (*Peking University*) . . . . Sep. 28th (Mon), 14:30–15:20

Frequency decomposition techniques for the nonlinear Schrödinger equation

Some global well posed results are obtained for the (derivative) nonlinear Schrödinger equations by using the frequency decomposition techniques.

Keywords. (Derivative) Nonlinear Schrödinger equation, global well posedness, rough data.

Hua Zhang (*North China University of Technology*) . . . Sep. 30th (Wed), 14:30–15:20

Global well-posedness and scattering for the fourth order nonlinear Schrödinger equations with small data

We show that the Cauchy problem for the fourth order nonlinear Schrödinger equations with the third order derivative nonlinear terms is globally well-posed in the modulation spaces $M^s_{2,1}(\mathfrak{R}^n)$.