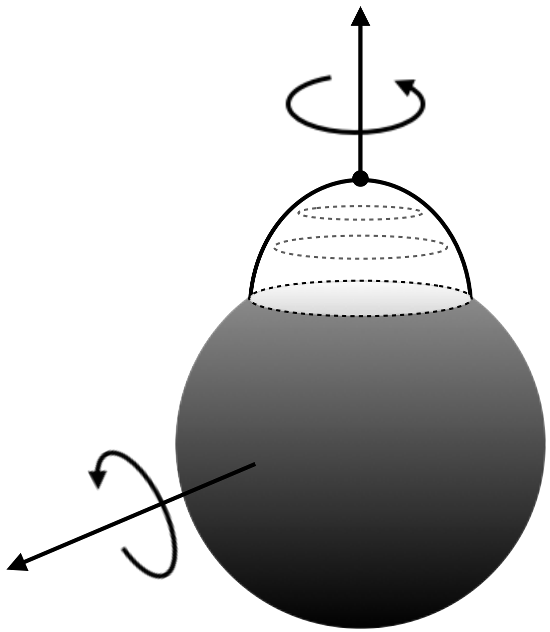
A Capped Black hole in Five Dimensions



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Based on 2311.11653

「一般相対論と幾何」研究会, 名古屋大学, 2024年2月8-9日

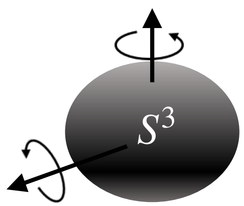
Introduction

モチベーション：各次元(特に5次元以上)にどのようなブラックホールがあり得るか？

漸近平坦な定常ブラックホールのホライズン面のトポロジー

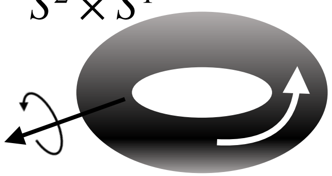
- 4次元定常ブラックホール：S^2 Hawking (1972), Chrusciel-Wald (1994)
- 5次元定常軸対称ブラックホール：S^3 or $S^2 \times S^1$ or $L(p,q)$

Galloway, Schoen (2006)
Hollands, Yazadjiev (2007)



Myers-Perry (1986)

$S^2 \times S^1$



Black ring

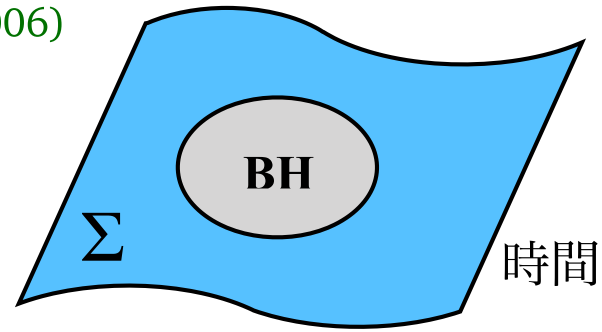
Emparan-Reall (2002)
Pomeransky-Sen'kov (2006)

…and more…

ブラックホールの外部領域の空間トポロジー

- 既知の真空解は全て $\simeq \mathbb{R}^n \backslash BH$

→ BHを除くと穴のないEuclid空間



時間一定面

$\Sigma \backslash \mathrm{BH} \simeq \mathbb{R}^n \backslash \mathrm{BH}$

これまで研究：どのようなトポロジーのホライズン面があり得るか？

今回着目すること：どのようなトポロジーの外部領域があり得るか？

Black hole Uniqueness

漸近平坦な定常BHの唯一性定理

漸近的な物理量（質量M, 角運動量J,電荷Q,...) → 解が唯一に決まる

BH is unique in D=4

漸近平坦かつ定常な真空中のblack hole ⇒ Kerr black hole with (M,J)

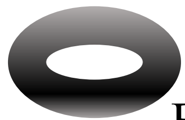
Carter 1972, Robinson 1975

BH is NOT unique in D=5

漸近平坦かつ定常軸対称な真空中の5次元BH

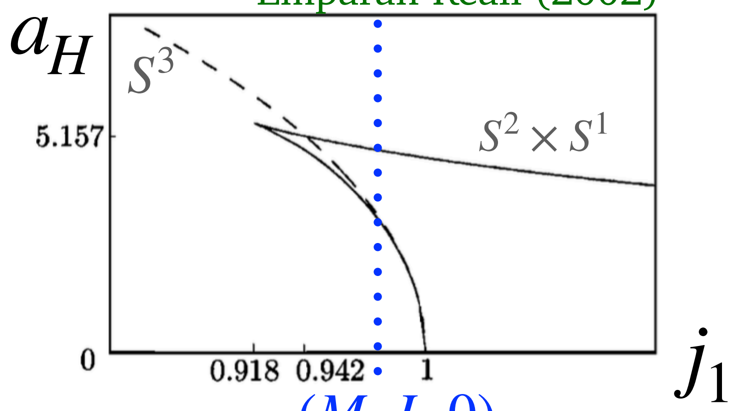
Ex) 同じ(M, J_1, J_2)を持つ球状BHとBlack ringが存在

Myers-Perry BH 

 $S^2 \times S^1$

Black ring

Emparan-Reall (2002)



$(M, J_1, 0)$

ホライズン形状を指定すれば唯一性定理が存在

Ex) Myers-Perry BH is unique solution for S^3-horizon　Morisawa-Ida (2004)

ただし、外部領域 $\simeq \mathbb{R}^4 \backslash$BH を仮定

→ **外部領域の仮定なしに唯一性が言えるか？**

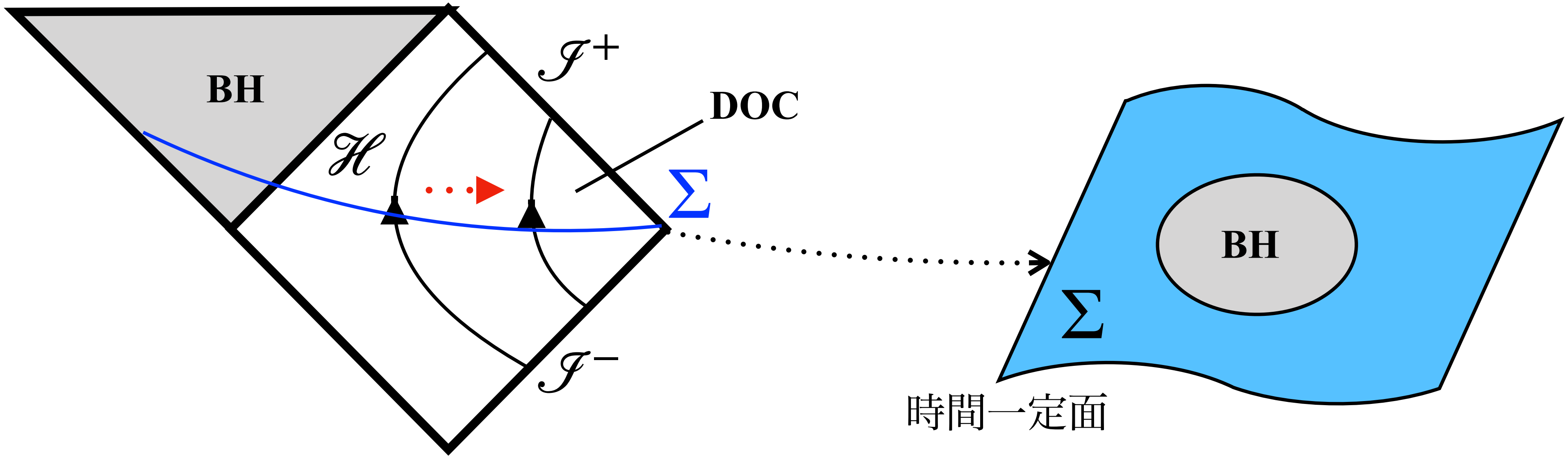
ブラックホール外部の
トポロジー

位相検閲定理

Topological Censorship Theorem Friedman-Schleich-Witt 1993

$\mathscr{I}^-$ から $\mathscr{I}^+$ へ向かう任意の2つの因果的な曲線はホモトピック

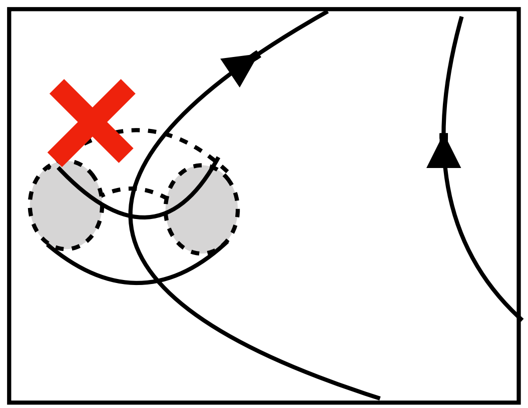
$\Leftrightarrow$ ブラックホールの外部領域 (Domain of outer communication (DOC))は単連結



4次元時空では非常に強い帰結

$\Rightarrow$ **DOCは自明なものに限る i.e.** $\mathrm{DOC} \cap \Sigma = \mathbb{R}^3 \backslash \mathbb{B}^3$

$\Rightarrow$ **ホライズン面の位相は** S^2 Chrusciel-Wald 1994



位相検閲定理

Topological Censorship Theorem Friedman-Schleich-Witt 1993

$\mathscr{I}^-$ から $\mathscr{I}^+$ へ向かう任意の2つの因果的な曲線はホモトピック

$\Rightarrow$ ブラックホールの外部領域 (Domain of outer communication (DOC))は単連結

5次元では弱い制限

連結和

$$\Rightarrow \mathrm{DOC} \cap \Sigma \simeq \mathbb{R}^4 \# n(S^2 \times S^2) \# m(\pm CP^2) \backslash \mathrm{BH}$$

Hollands-Holland-Ishibashi (2010)

非自明な外部構造　$n, m = 0, 1, \ldots$

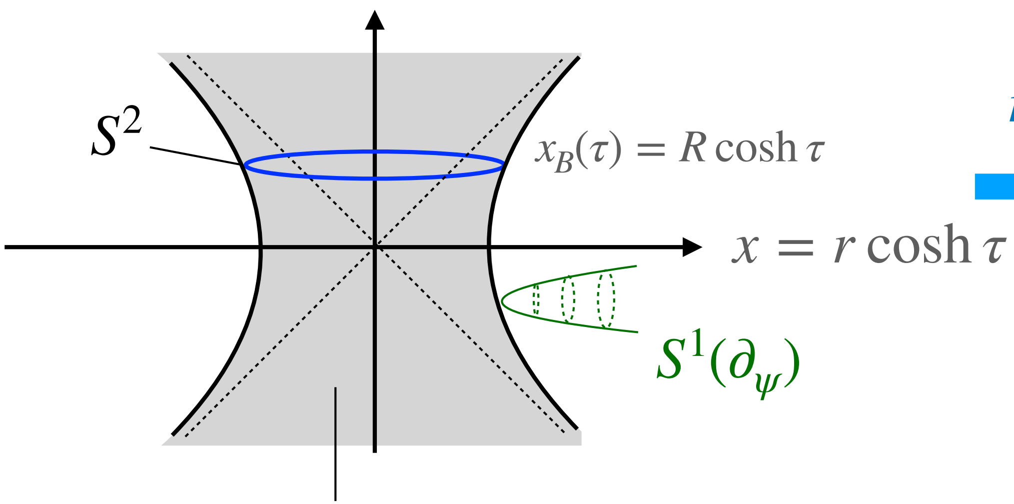
非自明な外部構造($\mathrm{DOC} \cap \Sigma \not\simeq \mathbb{R}^4 \backslash \mathrm{BH}$)を持ったBHがありえる？

Bubble of Nothing

Kaluza-Klein Bubble of Nothing Witten 1982

$$ds_5^2 = \left(1 - \frac{R^2}{r^2}\right) d\psi^2 + \frac{dr^2}{1 - \frac{R^2}{r^2}} - r^2 d\tau^2 + r^2 \cosh^2 \tau d\Omega_2^2$$

← Double Wick rotation from D=5 Schwarzschild



$r \gg R$

漸近的Kaluza-Klein時空

$$M_4 \times S^1 \quad \begin{cases} t = r \sinh \tau \\ x = r \cosh \tau \end{cases}$$

$$ds_5^2 \simeq d\psi^2 - dt^2 + dx^2 + x^2 d\Omega_2^2$$

何もない空洞（= Bubble of Nothing)が膨張していく

Bubble as Gravitational Solitons

磁場を入れることでBubbleが定常に存在できる

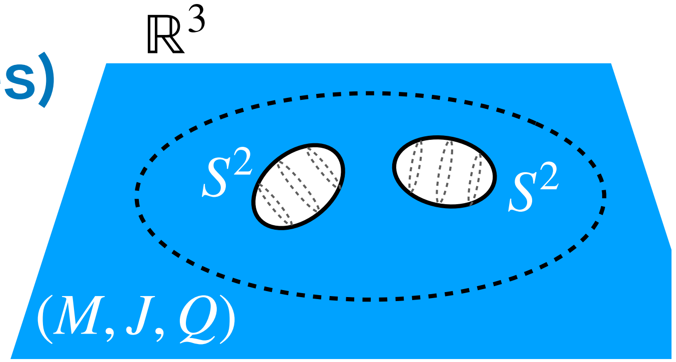
Gravitational Solitons (Microstate geometries)

Supersymmetric bubbles Bena-Warner (2008)

Non-supersymmetric bubbles

Bena-Gisto-Ruef-Warner (2009), Compere-Copey-Buyl-Mann (2009)

Bobev-Ruef (2010)



擬似BH

Black hole + bubbles ?

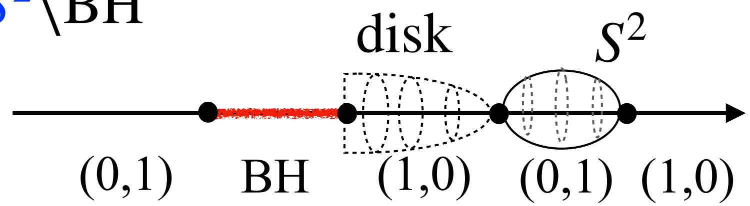
(少なくとも）時空が超対称ならば共存可能

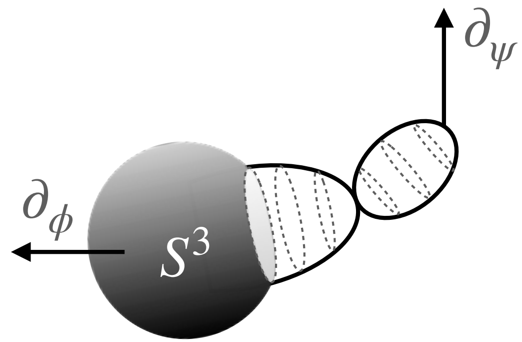
漸近平坦, 定常軸対称 S^3ホライズン + 2 bubbles

Kunduri-Lucietti 2014

$$DOC \cap \Sigma \simeq \mathbb{R}^4 \# S^2 \times S^2 \backslash \mathrm{BH}$$

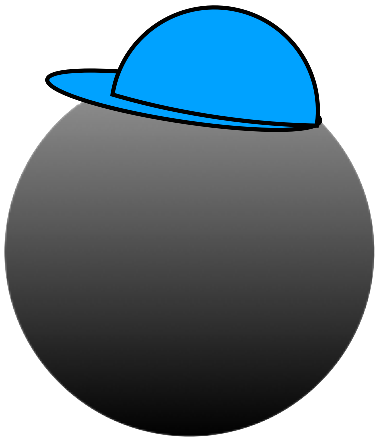
Rod構造→





より一般に（特に超対称でない）bubbleと共存するBHは存在するか？

Capped black hole



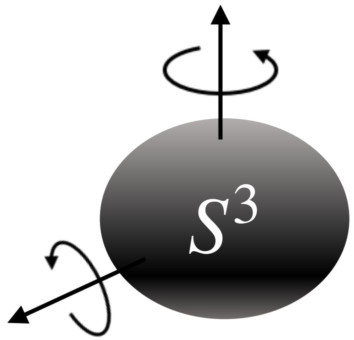
Spherical BH in D=5 Einstein-Maxwell-Chern-Simons

Bubble is supported by magnetic flux → consider **Charged BH**

D=5 Einstein-Maxwell-Chern-Simons

$$S = \frac{1}{16\pi G_5}\left[\int \sqrt{-g}\, d^5x \left(R - \frac{1}{4}F^2\right) - \frac{1}{3\sqrt{3}}\int F \wedge F \wedge A\right]$$

= bosonic sector of D=5 minimal SUGRA



(M, J_1, J_2, Q)

Cvetic-Youm BH Cvetic-Youm (1996)

The most general charged rotating AF BHs with S^3-horizon and DOC $\cap\, \Sigma \simeq \mathbb{R}^4 \setminus \mathbb{B}^4$

Uniqueness by Tomizawa-Yasui-Ishibashi (2009)

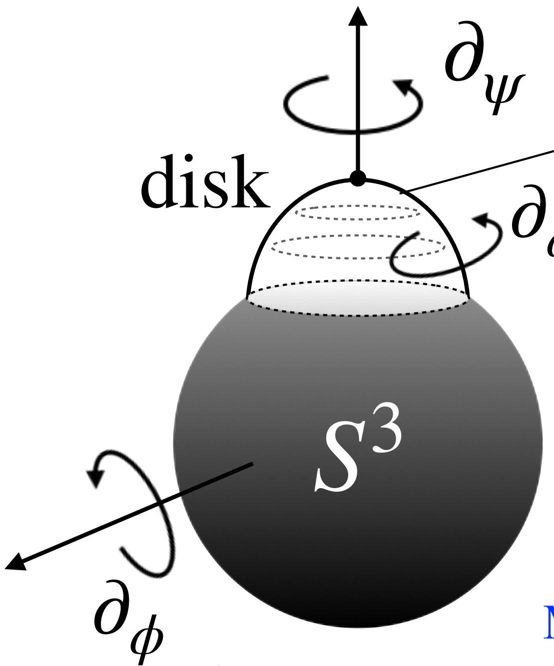
BPS limit → **BMPV BH**
Brekenridge-Myers-Peet-Vafa (1996)

Neutral limit → **Myers-Perry BH**
Myers-Perry (1986)

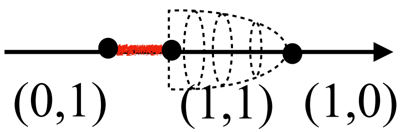
Static limit → **Reissner-Nordstrom BH in D=5**

Topology of new black hole

"Capped BH"



Rod 構造



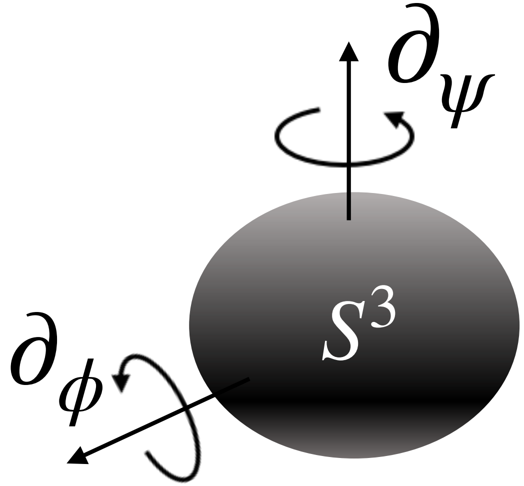
Magnetic flux

$(M, J_\psi, J_\phi, Q, q)$

$$\mathrm{DOC} \cap \Sigma \simeq \mathbb{R}^4 \# CP^2 \setminus \mathbb{B}^4$$

Orlik, Raymond (1970)
Khuri, Matsumoto, Weinstein, Yamada (2018)

Cvetic-Youm



(M, J_ψ, J_ϕ, Q)

$$\mathrm{DOC} \cap \Sigma \simeq \mathbb{R}^4 \setminus \mathbb{B}^4$$

Cf) $\mathrm{DOC} \cap \Sigma \simeq \mathbb{R}^4 \# n(S^2 \times S^2) \# m(\pm CP^2) \setminus \mathrm{BH}$
Hollands-Holland-Ishibashi (2010)

$G_{2(2)}/SL(2,R)\times SL(2,R)$ Nonlinear σ-model

定常かつ軸対称な時空を仮定（Killing vectors $\xi_i = (\partial_t, \partial_\psi, \partial_\phi)$ ）

$(g_{\mu\nu}, A_\mu)$ in 5次元EMCS理論（超重力理論）with two Killing vectors $(\partial_t, \partial_\psi)$

→ $G_{2(2)}$ 対称性

Reduce (t, ψ)

Nonlinear sigma model of $G_{2(2)}/SL(2,R)\times SL(2,R)$

Theory of 8 scalar functions on 3D

$$\Phi_A := (\lambda_{00}, \lambda_{01}, \lambda_{11}, \omega_0, \omega_1, \mu, \psi_0, \psi_1)$$

(Harmonic map from (ρ, z, ϕ))

Mizoguchi-Ohta (1998)

$$\begin{cases} \lambda_{ij} := \xi_{i,\mu}\xi_j{}^{\mu}, \quad (i,j=0,1) \\ \omega_0,\ \omega_1 \ \text{: Twist potentials} \\ \mu, \psi_0, \psi_1 \ \text{: Electromagnetic potentials} \end{cases}$$

Harrison 変換 $\in SL(2,R)\times SL(2,R) \subset G_{2(2)}$

Bouchareb-Clement-Chen-Gal'tsov-Scherbluk-Wolf (2007)

漸近平坦性を保ったまま電場や磁場を作る変換

Construction

逆散乱法 + Harrison変換

Belinsky, Sakharov (1979)
Pomeransky (2006)

metric+gauge potential

A vacuum “seed” from 逆散乱法

(Singular rotating black lens)

Chen-Teo (2008) $(g_{\mu\nu}, A_\mu = 0)$

1

target space : $G_{2(2)}/SL(2,R) \times SL(2,R)$ coset space

$\Phi_A := (\lambda_{00}, \lambda_{01}, \lambda_{11}, \omega_0, \omega_1, 0, 0, 0)$

2 **Harrison変換**

$\Phi'_A := (\lambda'_{00}, \lambda'_{01}, \lambda'_{11}, \omega'_0, \omega'_1, \mu', \psi'_0, \psi'_1)$

3

New charged black hole

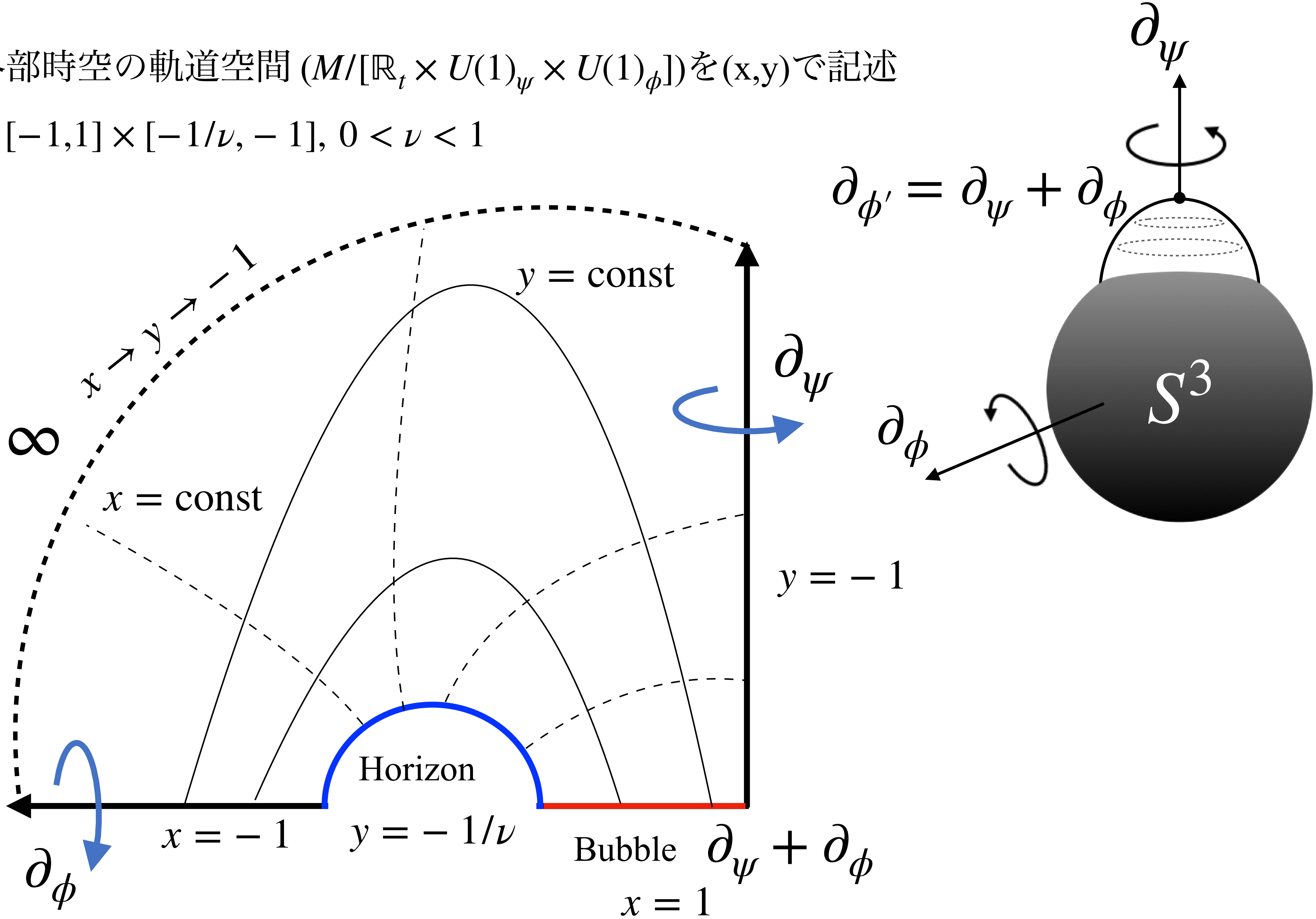
$(g'_{\mu\nu}, A'_\mu)$

電磁場ポテンシャル

C-metric 計量

BHの外部時空の軌道空間 $(M/[\mathbb{R}_t \times U(1)_\psi \times U(1)_\phi])$を(x,y)で記述

$(x, y) \in [-1,1] \times [-1/\nu, -1],\ 0 < \nu < 1$



Exact solution for metric and gauge potential

5D metric in C-metric form

$(-1 \leq x \leq 1,\ -1/\nu \leq y \leq -1)$

$$ds^2 = -\frac{H(y,x)}{D^2 H(x,y)}(dt + \Omega'_\psi d\psi + \Omega'_\phi d\phi)^2 + D\left[\frac{F(y,x)}{H(y,x)}d\psi^2 - \frac{2J(x,y)}{H(y,x)}d\psi d\phi - \frac{F(x,y)}{H(y,x)}d\phi^2\right] + \frac{\ell^2 D H(x,y)}{4(1-\gamma)^2(1-\nu)^2(1-a^2)(x-y)^2}\left(\frac{dx^2}{G(x)} - \frac{dy^2}{G(y)}\right)$$

Gauge potential

$$A = \frac{\sqrt{3}cs}{DH(x,y)}\left[\left\{H(x,y) - H(y,x)\right\}dt - \left\{cH(y,x)\Omega_\psi(x,y) - sH(x,y)\Omega_\phi(y,x)\right\}d\psi - \left\{cH(y,x)\Omega_\phi(x,y) - sH(x,y)\Omega_\psi(y,x)\right\}d\phi\right]$$

With $D = \frac{c^2 H(x,y) - s^2 H(y,x)}{H(x,y)}$, $\Omega'_\psi = c^3\Omega_\psi(x,y) - s^3\Omega_\phi(y,x)$, $\Omega'_\phi = c^3\Omega_\phi(x,y) - s^3\Omega_\psi(y,x)$

and $(c, s) := (\cosh\alpha, \sinh\alpha)$

$H, F, J, \Omega_\psi, \Omega_\phi$は(x,y)の有理式

Metric functions and parameters

解のパラメータ $(\ell, \nu, \gamma, a, b, \alpha)$

逆散乱法などの変換で作った解は一般に物理的な要請を満たさない

→ 時空が物理的であるようにパラメータを制限する

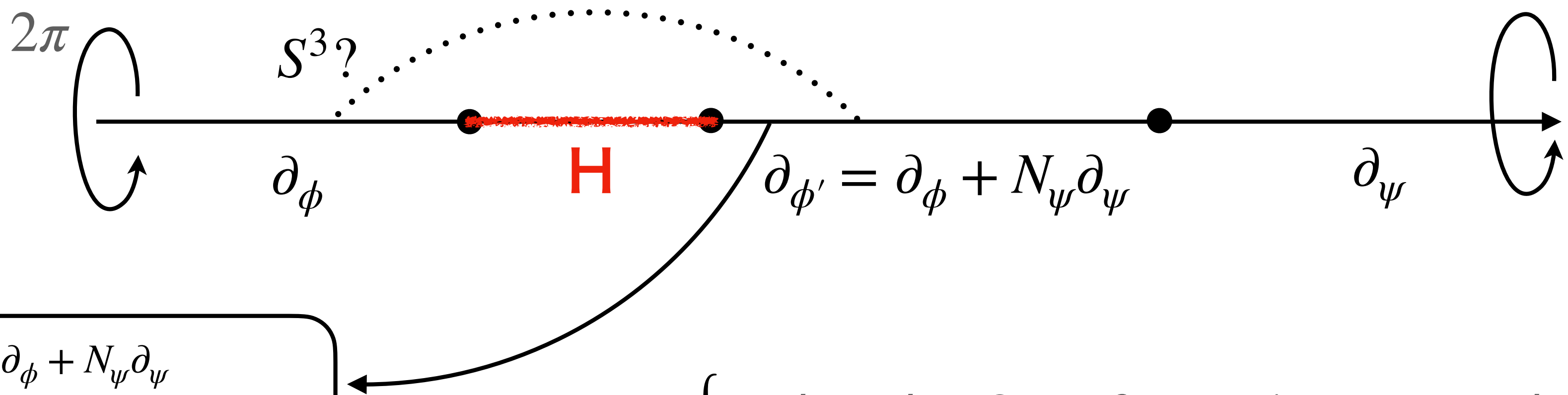
Functions

$$
\begin{aligned}
G(\xi) &= (1-\xi^2)(1-\nu\xi),\\
H(x,y) &= 2d_1(1+\nu)^{-1}(1-\gamma)(1-\nu)[2+\nu(1+x+y-xy)][\gamma(1+y)(1+\nu x)-2-\nu(3x+\nu+y(2+x+\nu+2x\nu))]\\
&+ d_1c_3[\gamma+\gamma\nu x-\nu(x+\nu)](1+x)(1+y)^2\\
&+ (1+\nu)^{-1}(1-\gamma)(1-\nu)^2(x+y+\nu+\nu xy)\Big[2\left((1-\gamma)(1-\nu)(\gamma+\nu)-2d_2\right)\left(2+\nu(1+x+y-xy)\right)\\
&+ \{2(\gamma-\nu)(2+(x+y)\nu)+(1-xy)((3-\nu)\nu-\gamma(1+\nu))-(1-\nu)(\gamma+\nu)(x-y)\}\,c_3\Big],\\
F(x,y) &= \frac{2\ell^2}{(1-a^2)(x-y)^2}\Big[4\left\{\left(1-a^2\right)^2(y-1)(1-\gamma)^3(1-\nu)^3-(1+y)d_1^2\right\}(1+y\nu)G(x)\\
&+ 4\left\{(1-\nu)c_2-(1-ab)(\gamma-\nu)(1+\nu)c_1\right\}^2(1+x\nu)(1+x)G(y)\\
&+ \nu^{-1}(1-\nu)^3(\gamma-\nu)\left\{d_3^2(1-x^2)G(y)-c_3^2(1-y^2)G(x)\right\}\\
&+ \frac{G(x)G(y)\left[\left(1-a^2\right)(1-\gamma)d_4-\nu c_2^2(a-b)^2(1-\gamma)^2(\gamma-\nu)y+\nu x(\gamma-\nu)\left(c_1c_3-bd_1\right)^2\right]}{\nu(1-\gamma)}\Big],\\
J(x,y) &= \frac{2\ell^2(1+x)(1+y)}{(1-a^2)(x-y)}\Big[4d_1\left\{(a-b)(1-\gamma)(\gamma-\nu)(1+\nu)-ad_2\right\}(1+\nu x)(1+\nu y)\\
&- d_3c_3(1-\nu)^3(\gamma-\nu)(1-x)(1-y)-c_2(a-b)(\gamma-\nu)\left(c_1c_3-bd_1\right)(1-x)(1-y)(1+x\nu)(1+y\nu)\Big],\\
\Omega_\psi(x,y) &= \frac{v_0\ell(1+y)(1-\nu)}{\nu H(y,x)}\Big[c_2\left(c_1c_3-bd_1\right)(1-x)(1+x\nu)(1+y\nu)\\
&- d_2c_3(1-\nu)^2(1-x)+d_1(1+x\nu)\left\{2\nu(1-ab)(1-\gamma)(1+\nu)(1+x)+c_3(1-3\nu-x(1+\nu))\right\}\Big],\\
\Omega_\phi(x,y) &= \frac{v_0\ell}{H(y,x)}\Big[d_1b(1+x)\left\{d_2(1+y)(1+y\nu)+\nu c_3\left(1-y^2\right)(1-\nu)\right\}\\
&+ \frac{2(a-b)(1-\gamma)^2\left\{2d_1(1+x\nu)(1+y\nu)^2-\nu c_3(1-\nu)^2(1-y)(x+y+\nu+xy\nu)\right\}}{1+\nu}\Big],
\end{aligned}
$$

Auxiliary params.

$$
\begin{aligned}
v_0 &:= \sqrt{\frac{2(\gamma^2-\nu^2)}{(1-a^2)(1-\gamma)}},\\
c_1 &:= a(1-\gamma)+b(\gamma-\nu),\\
c_2 &:= 2a\nu(1-\gamma)+b(\gamma-\nu)(1+\nu),\\
c_3 &:= 2\nu(1-\gamma)+b^2(\gamma-\nu)(1+\nu),\\
d_1 &:= c_1^2(\nu+1)-(1-\gamma)(1-\nu)^2,\\
d_2 &:= bc_1(\nu+1)(\gamma-\nu)+2\nu(1-\gamma)(1-\nu),\\
d_3 &:= b(1-a^2)(1-\gamma)(\nu+1)-ac_3,\\
d_4 &:= b^2(\gamma-\nu)\left[c_1^2(\nu+1)^2\left(1-\nu^2-3(1-\gamma)\nu\right)\right.\\
&\left.-(1-\gamma)(1-\nu)^4(2\nu+1)\right]+(1-\gamma)\left[\left(c_2(1-\nu)\right.\right.\\
&\left.\left.-2\nu^2c_1\right)^2-4\nu^2c_1^2\left(3\nu^2+1-\gamma(\nu+2)\right)\right].
\end{aligned}
$$

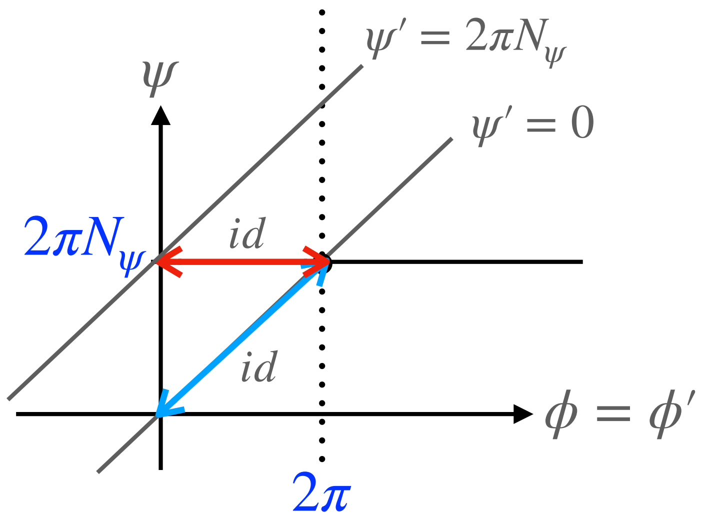
Topology condition



$$\partial_{\phi'} = \partial_\phi + N_\psi \partial_\psi$$
$$\partial_{\psi'} = \partial_\psi$$
$$\Rightarrow \ \psi' = \psi - N_\phi \phi, \ \phi' = \phi$$

$$\begin{cases} \phi \sim \phi + 2\pi \quad \text{for} \quad \psi' = \psi - N_\psi \phi = \text{const} \\ \phi \sim \phi + 2\pi \quad \text{for} \quad \psi = \text{const} \end{cases}$$

$$\Rightarrow \begin{cases} \psi' \sim \psi' + 2\pi N_\psi \\ \psi \sim \psi + 2\pi N_\psi \end{cases}$$



同時に $\psi \sim \psi + 2\pi$ for $\phi = \text{cosnt}$ でもある

$$\Rightarrow \ N_\psi = 0, \pm 1, \pm 2, \ldots$$

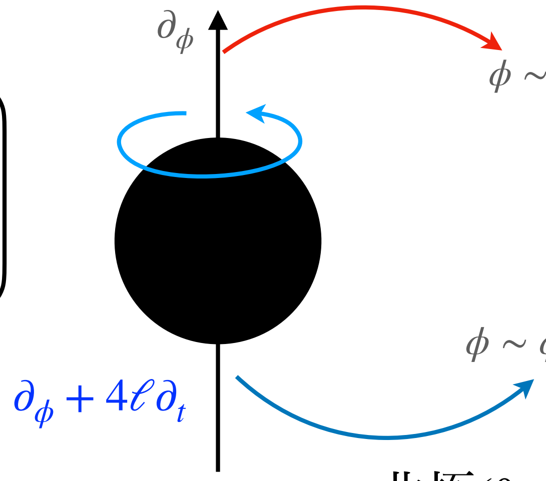
ホライズン面のトポロジー $\begin{cases} N_\psi = 0 : S^2 \times S^1 \\ N_\psi = \pm 1 : S^3 \\ N_\psi = \pm p, \ (p \geq 2) : \text{L(p,1)} \end{cases}$

Dirac-Misner string singularity
Misner (1963)

Taub-NUT black hole
Taub (1951), Newman-Tamburino-Unti (1963)

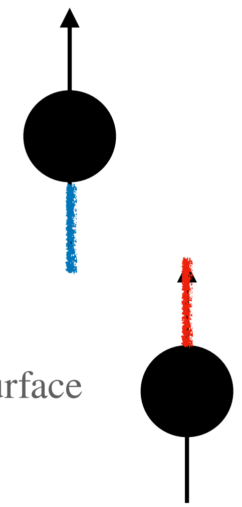
$$ds^2 = -f(r)[dt + 4\ell \sin^2(\theta/2) d\phi]^2 + f(r)^{-1} dr^2 + (r^2 + \ell^2)(d\theta^2 + \sin^2\theta d\phi^2)$$

$$f = 1 - \frac{2(mr + \ell^2)}{r^2 + \ell^2}$$



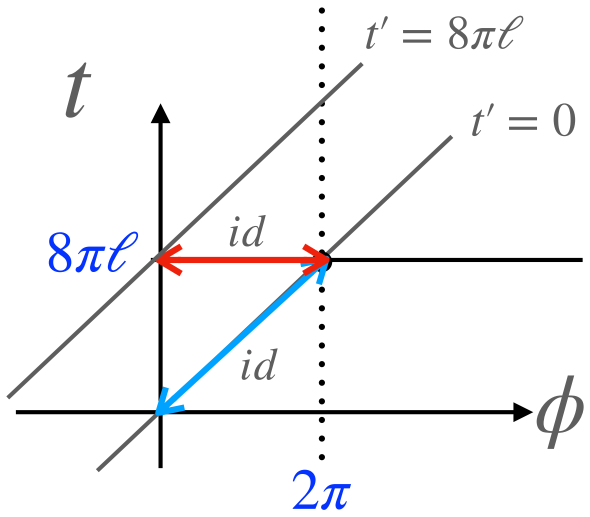
$\phi \sim \phi + 2\pi$ on $t = C$ surface

$\phi \sim \phi + 2\pi$ on $t' = t - 4\ell\phi = C$ surface



北極(θ=0)か南極（θ=π/2)にConical Singularity

・Conicalを回避するため、両方の軸で周期性を課す



$$\begin{cases} \phi \sim \phi + 2\pi \quad \text{for} \quad t = \text{const} \\ \phi \sim \phi + 2\pi \quad \text{for} \quad t' = t - 4\ell\phi = \text{const} \end{cases}$$

$$\Rightarrow t \sim t + 8\pi\ell$$

Conical Singularity or 時間が周期的

Regularity conditions

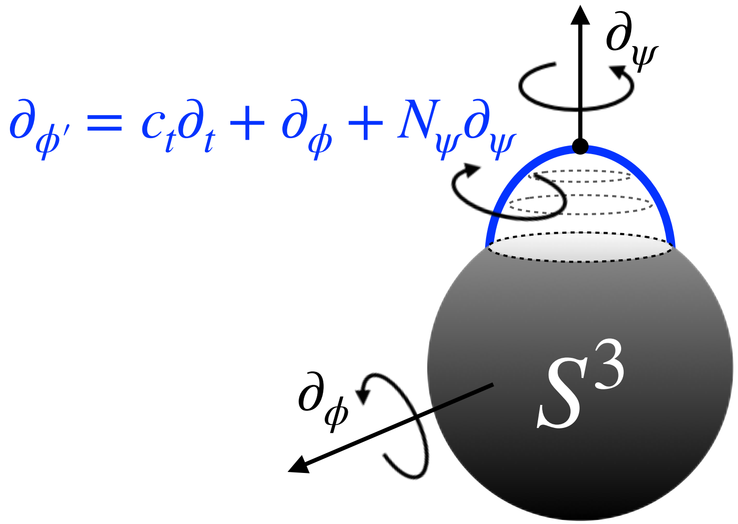
解の6つのパラメータ $(\ell, \nu, \gamma, a, b, \alpha)$ に物理的条件を課す

Bubble上の正則条件

1. No Dirac-Misner string singularity

$c_t \neq 0 \Rightarrow$ 特異点 or 時間の周期性

$$c_t = 0 \Leftrightarrow \tanh^3 \alpha = \frac{a-b}{1-ab}$$



2. Topology condition

$$1 = N_\psi = \frac{a d_1 + (1-\gamma)(1+\nu)(1-a^2)c_1}{d_1}$$

3. No Conical singularity

$$1 = \left(\frac{\Delta\phi'}{2\pi}\right)^2 = \frac{d_1^2}{\left(1-a^2\right)^2 (1-\gamma)^3 (1-\nu)^2 (1+\nu)}$$

条件1.2.3.
→ 曲率特異点, CTCも存在しない

物理的な解：独立なパラメータは3つ

Parameter space

正則性を課すと独立なパラメータは(ℓ, α, ν)の3つ

- **Scaling :** $\ell > 0$

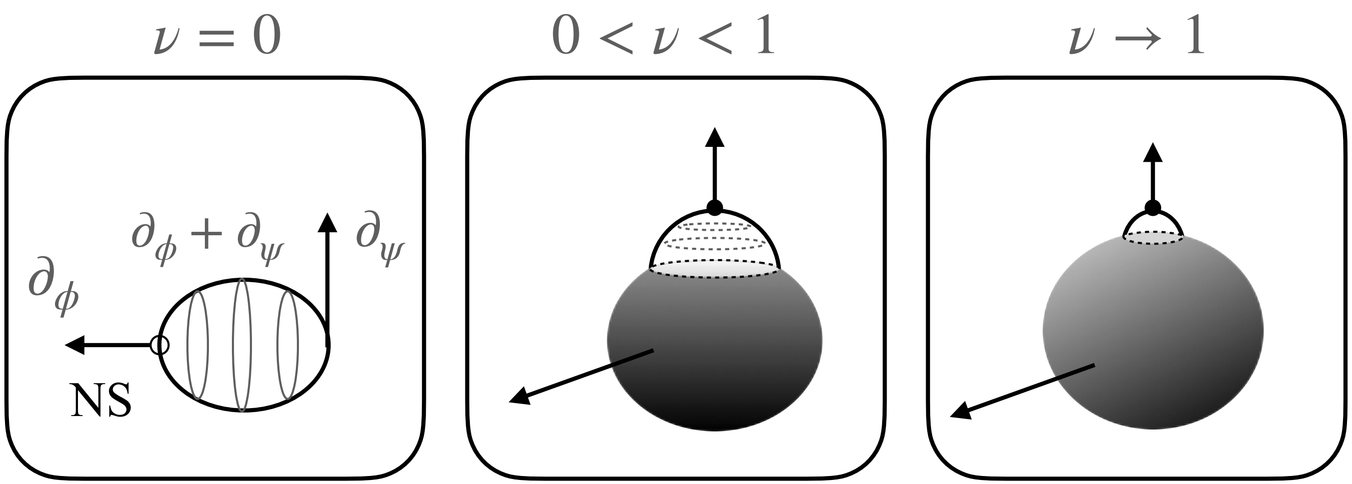
- **Charge parameter :** $\alpha = 0 \sim \infty$

$$\frac{Q}{M} = \frac{2}{\sqrt{3}} \tanh(2\alpha)$$

$\alpha \to 0$: 特異な真空計量

$\alpha \to \infty$: BPS(supersymmetric) limit　（特異な計量）

- **Moduli of horizon and bubble :** $\nu = 0 \sim 1$

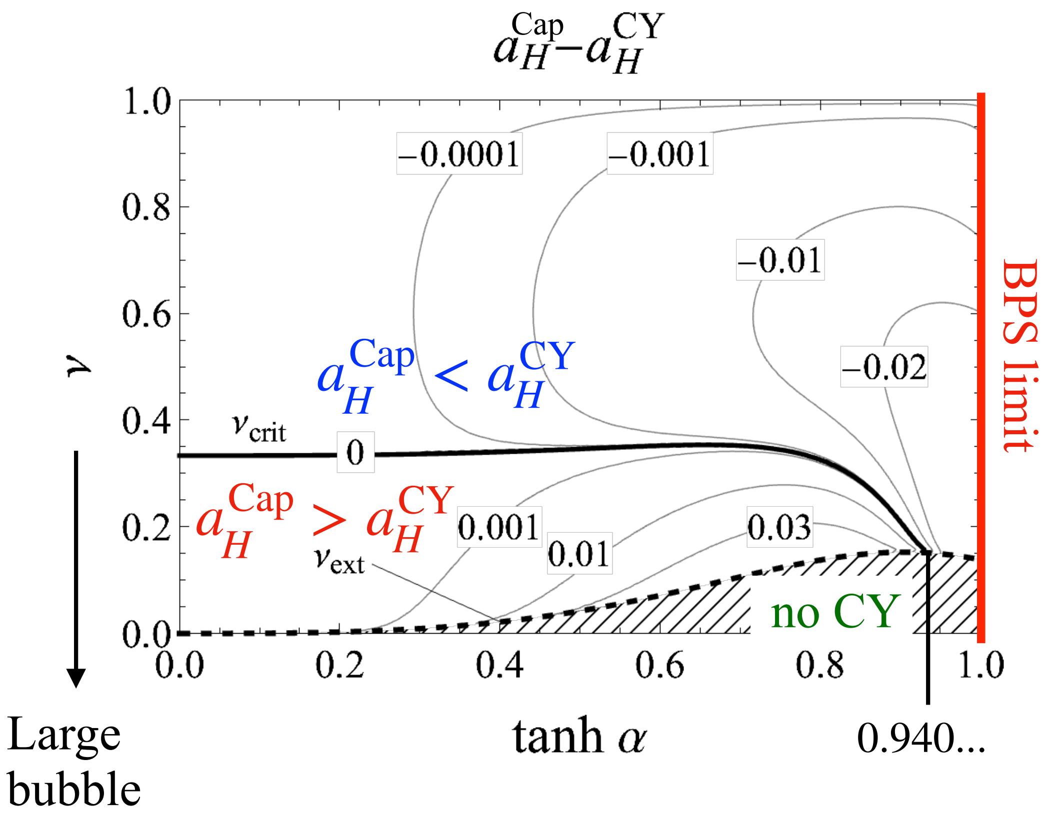


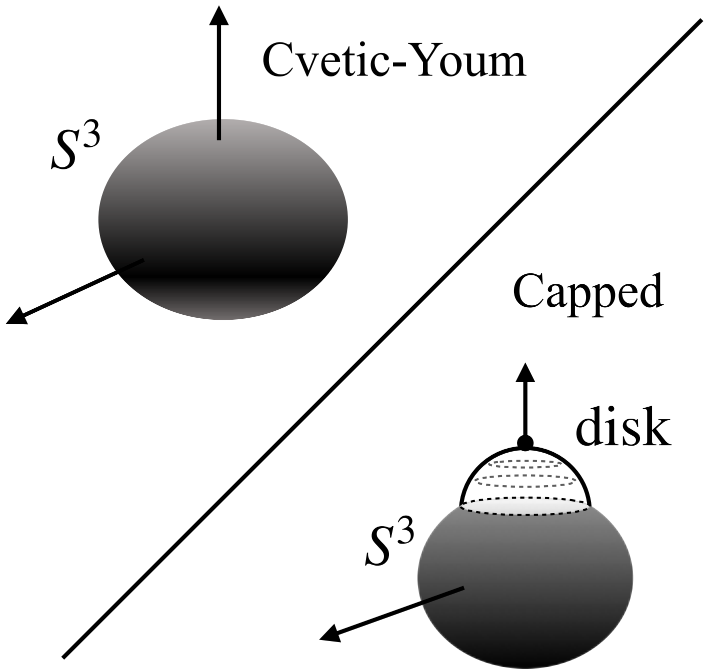
Bubble >> BH
at $\nu \to 0$
Bubble << BH
at $\nu \to 1$

$\nu = 0{,}1$で特異

Non-uniqueness for Spherical black holes

質量あたりのホライズン面積(エントロピー)を比較 ：Capped v.s CY





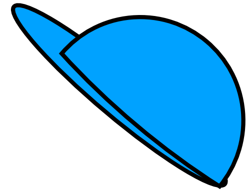
- $\nu_{\rm ext}(\alpha) \le \nu \le 1$においてCapped BHと$(M, J_\psi, J_\phi, Q)$を持つCY BHが存在

  → S^3-ホライズンに関する唯一性の破れ

- $\nu_{\rm ext}(\alpha) \le \nu < \nu_{\rm crit}$においてはCapped BHの方が面積が大きい

  → **Capped BHがより安定**

Summary

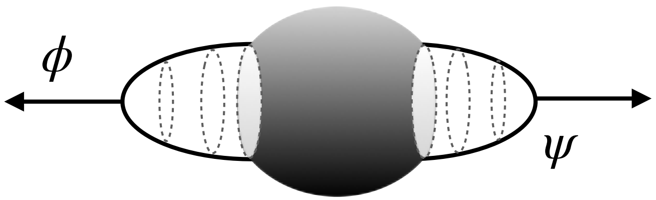


- 5次元Einstein-Maxwell-Chern-Simons理論(D=5 minimal SUGRA)において非自明な外部構造(disk状のbubble)を持つ球状BH (Capped BH)を求めた
- 得られた時空には特異点がなく、CTCも存在しない

- Capped BHは同じS^3ホライズンを持つ解(Cvetic-Youm BH)と同じ(M, J_ψ, J_ϕ, Q)が持てる

  → S^3-ホライズンに関する唯一性が破れている

  → bubbleが十分に大きいとき、Capped BHの方が安定になる



BH+More bubbles？