QEDの赤外発散と漸近対称性

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平井隼人氏 (木更津高専) との共同研究に基づく

[1901.09935], [2009.11716], [2209.00608]

KMI分野横断セミナー「時空の漸近構造、赤外発散、重力波」, 12月18日, 2022

Introduction

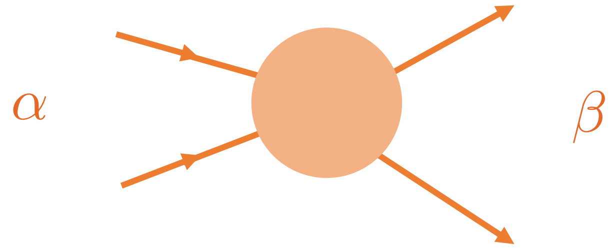
- 重力理論における漸近対称性と低エネルギーの物理 [Strominger, ...]
  - 重力波のメモリー効果
  - soft graviton theorem

- 電磁気学でも同様の関係
  - 電磁波のメモリー効果
  - soft photon theorem

■ 赤外発散

- 量子電磁気学(QED)における赤外発散

    - 散乱のS行列 $S_{\beta,\alpha} = \langle\beta|\, S \,|\alpha\rangle$



一般に摂動計算のループレベルで発散

- 紫外発散 (高エネルギー)
- **赤外発散 (低エネルギー)**

- 最近の理解: 赤外発散は漸近対称性と密接に関連している.

    - QEDの漸近対称性を保証するには赤外発散が不可欠.
    - メモリー効果を量子論で実現するにはドレス状態が必要.

Outline

1. Introduction

2. Asymptotic symmetry in QED

3. IR divergences in QED and dressed states

4. Dressed states and asymptotic symmetry

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Asymptotic symmetry in QED

- Gravitational theory on asympt flat spacetime has infinite-dim asympt sym (BMS symmetry).

- Electromagnetic theory also has an infinite dim sym related to asymptotic behaviors of fields. [He, Mitra, Porfyriadis, Strominger (2014), (2015)]

- QED has a U(1) gauge symmetry: $\delta A_\mu(x) = \partial_\mu \epsilon(x), \quad \delta\psi(x) = ie\,\epsilon(x)\psi(x)$

  Parts of them are physical symmetry (not gauge redundancy).

  e.g., global trsf. $\epsilon = const.$

- This symmetry leads to the electromagnetic memory effect.

  [Hirai, SS (2018)]

Infinite asymptotic symmetry

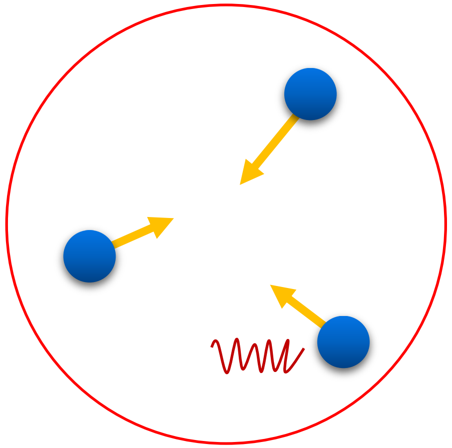
- Local U(1) transformations: $\delta A_\mu(x) = \partial_\mu \epsilon(x), \quad \delta\psi(x) = ie\,\epsilon(x)\psi(x)$
- Charge of local U(1) trsf: $Q[\epsilon] = \int d^3x\,\partial_i(F^{0i}\epsilon)$

If $Q[\epsilon] = 0$ for any configuration of fields, the trsf is gauge redundancy.
Not the case for $\epsilon(x)$ which doesn't decay at the asymptotic region (large trsf).

- Asymptotic symmetry is physical sym.

- D.o.f of symmetry is a function of two-sphere $\lim_{r\to\infty} \epsilon(x) = \epsilon^{(0)}(\theta, \varphi)$

⇨ Infinite dim physical symmetry

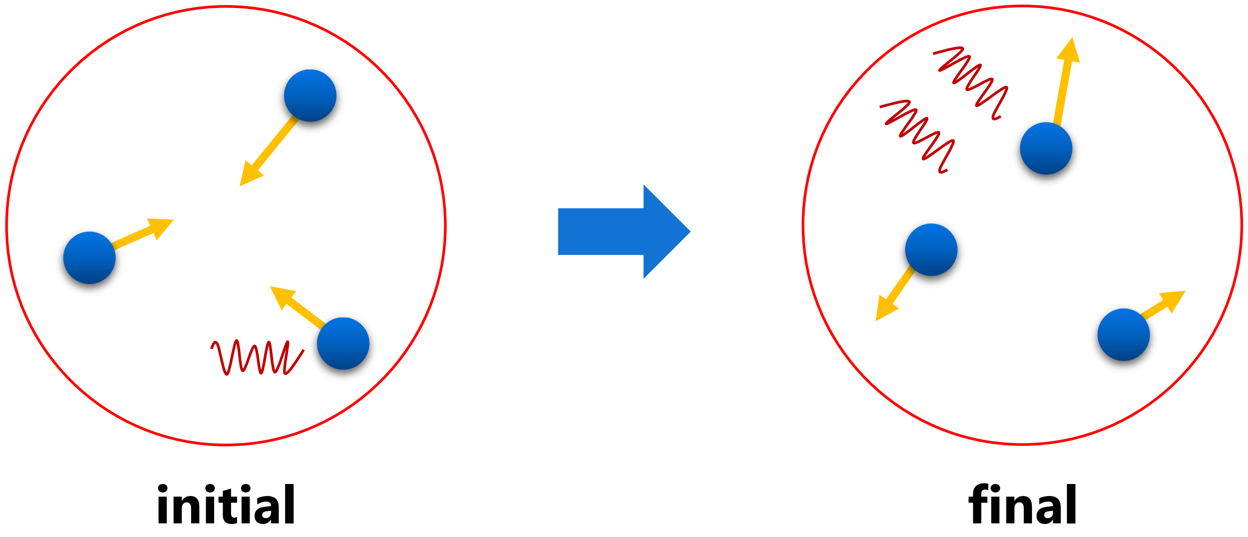


Conservation law for asymptotic symmetry

- Symmetry $\Rightarrow$ Conservation law $\partial_\mu J^\mu = 0$ $\quad (J^\mu = F^{\mu\nu}\partial_\nu \epsilon + j^\mu_{\rm mat}\epsilon)$

  Infinite number of conserved charges $Q_{\rm in}[\epsilon] = Q_{\rm fin}[\epsilon]$

$Q_{\rm in}[\epsilon]$



$Q_{\rm fin}[\epsilon]$

- $\epsilon^{(0)} = \text{const.}$ $\Rightarrow$ Conservation of electric charges
- $\epsilon^{(0)}(\theta, \varphi)$ $\Rightarrow$ Other conservation laws (memory effect)

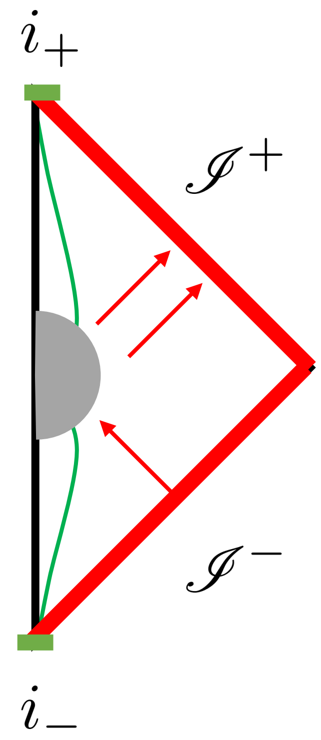
Conserved quantity

$$Q[\epsilon] = Q^{\mathrm{hard}}[\epsilon] + Q^{\mathrm{soft}}[\epsilon]$$

$$Q^{\mathrm{hard}}_{\mathrm{in}}[\epsilon] = \int_{S^2(t=-\infty)} d\mathbf{S}\cdot\mathbf{E}\,\epsilon^{(0)}$$ "electric multiple moment"

For general $\epsilon(\theta,\varphi)$, $Q^{\mathrm{hard}}_{\mathrm{in}}[\epsilon] \neq Q^{\mathrm{hard}}_{\mathrm{fin}}[\epsilon]$

$$Q^{\mathrm{soft}}_{\mathrm{in}}[\epsilon] = \int_{\mathscr{I}^-} dV\, F^{rA}\partial_A \epsilon$$ "incoming electromagnetic waves"



- Current conservation $\partial_\mu J^\mu = 0$ $\quad (J^\mu[\epsilon] = F^{\mu\nu}\partial_\nu \epsilon + j^\mu_{\mathrm{mat}}\epsilon)$

$\Rightarrow$ $Q_{\mathrm{in}}[\epsilon] = Q_{\mathrm{fin}}[\epsilon]$

This conservation law is nothing but the memory effect. [Hirai & SS (2018)]

Memory effect as a conservation law

- Conservation law $Q_{\rm in}^{\rm hard}[\epsilon] + Q_{\rm in}^{\rm soft}[\epsilon] = Q_{\rm fin}^{\rm hard}[\epsilon] + Q_{\rm fin}^{\rm soft}[\epsilon]$

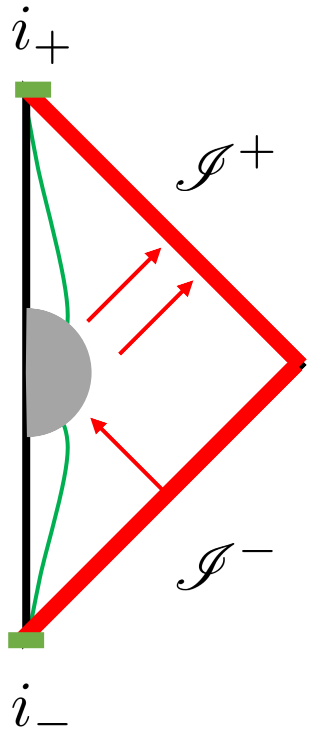
In Lorenz gauge $\partial_\mu A^\mu = 0$, $A_B(u,r,\Omega) = A_B^{(0)}(u,\Omega) + \mathcal{O}(1/r)$

angular component $(u = t - r)$

$$Q_{\rm fin}^{\rm soft}[\epsilon] = \int_{S^2} d^2\Omega\, \delta A_B^{(0)} \partial^B \epsilon^{(0)} , \quad \delta A_B^{(0)} = A_B^{(0)}(u=\infty) - A_B^{(0)}(u=-\infty)$$

$$\int_{S^2} d^2\Omega\, \delta A_B^{(0)} \partial^B \epsilon^{(0)} = Q_{\rm in}^{\rm hard}[\epsilon] - Q_{\rm fin}^{\rm hard}[\epsilon] + Q_{\rm in}^{\rm soft}[\epsilon]$$

memory effect



- There is a permanent shift of the final EM field, and the leading part of it is fixed by the change of the hard charges (and the initial EM field).

- electromagnetic analogue of the gravitational memory effect

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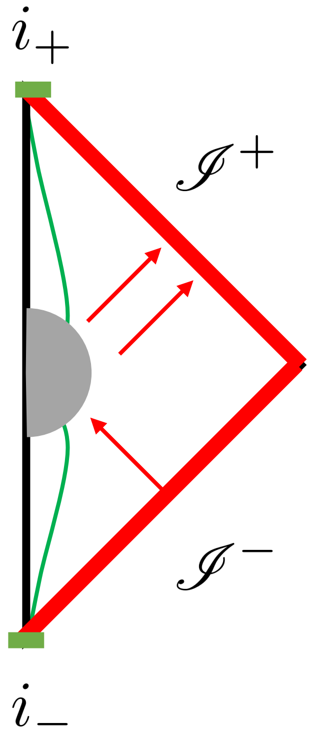
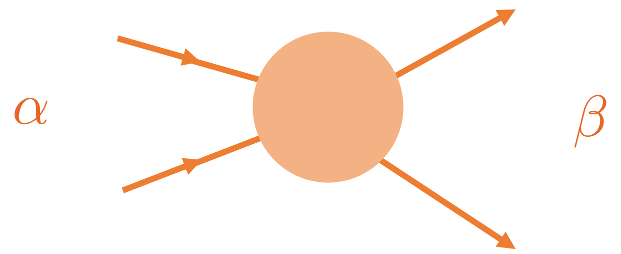
Scattering in QED



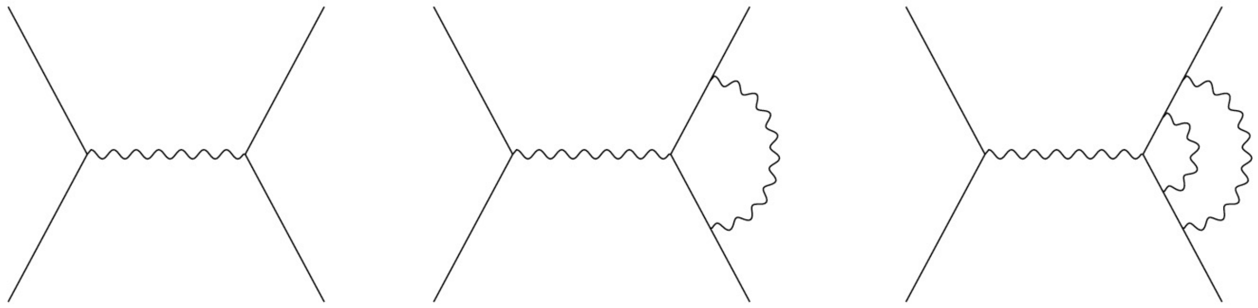
- QED describes quantum dynamics of electromagnetic waves and charged particles.

- The fundamental quantity is S-matrix.

$$S_{\beta,\alpha} = \langle\beta|\, S\, |\alpha\rangle$$

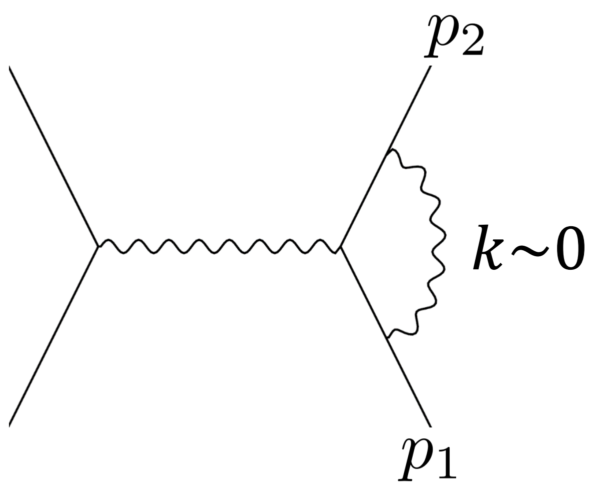


- Perturbatively computed by Feynman diagrams



Infrared divergences in QED

Loop integral may diverge by virtual soft photons



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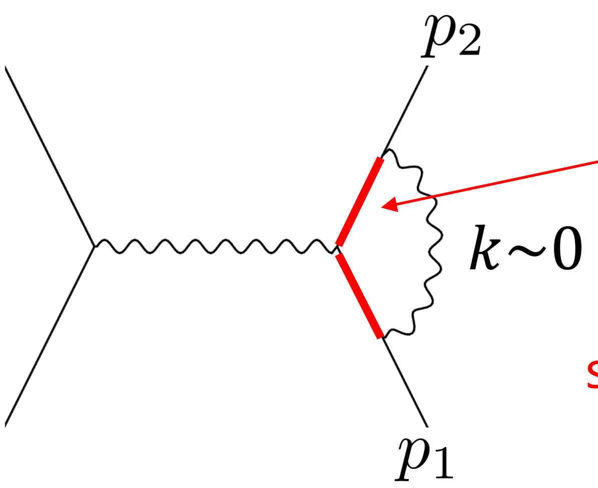


almost on-shell electron if the virtual photon is soft

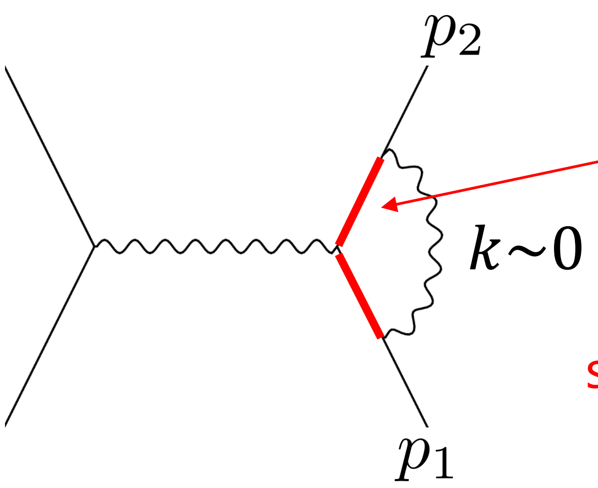
soft photon approx

$$\int \frac{d^3k}{(2\pi)^3(2\omega)} \frac{1}{p_1 \cdot k} \frac{1}{p_2 \cdot k} \sim \log \lambda$$

λ : IR cutoff

Infrared divergences in QED

Loop integral may diverge by virtual soft photons



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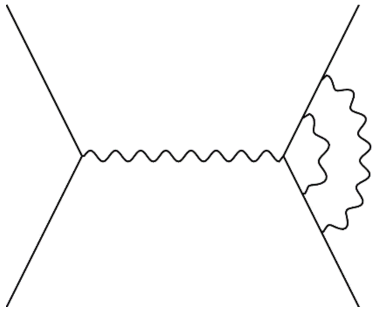


$\sim (\log \lambda)^2$

Factorization of IR divergences

We can sum up all of IR divergent parts. The divergent parts are factorized.

S-matrix element with IR cutoff λ: $$\boxed{S^{\lambda}_{\alpha,\beta} = \underbrace{\left(\frac{\lambda}{\Lambda}\right)^{A}}_{\text{soft}} \underbrace{S^{\Lambda}_{\alpha,\beta}}_{\text{hard}}}$$ Λ: new IR cutoff

The power A depends only on $\{e_n, p_n\}$ of initial and final charged particles and is independent of the details of scattering.

This IR factorization is valid as long as we can use the soft photon approx for $|\vec{k}| < \Lambda$.

We will say 'soft' for $\lambda < |\vec{k}| < \Lambda$ and 'hard' for $|\vec{k}| > \Lambda$.

No nontrivial scattering?

$$\boxed{S^{\lambda}_{\alpha,\beta} = \left(\frac{\lambda}{\Lambda}\right)^{A} S^{\Lambda}_{\alpha,\beta}}$$

The power $A > 0$ except for trivial process $\{e_n, p_n\}_\alpha = \{e_n, p_n\}_\beta$.

$\left(\frac{\lambda}{\Lambda}\right)^{A} \to 0$ if we remove the IR cutoff $\lambda \to 0$.

$$\boxed{\lim_{\lambda\to 0} S^{\lambda}_{\alpha,\beta} \to 0.}$$

<u>Any S-matrix elements are zero except for trivial diagonal elements???</u>

It seems that there is no nontrivial scattering…

This is the IR problem in QED.

Conventional treatment (inclusive formalism)

[Bloch and Nordsieck (1937)]

■ Any detector has a resolution E_d. Soft photons $k < E_d$ can't be detected.

- Detector can't distinguish $\beta \to \alpha, \quad \beta \to \alpha + \gamma_{soft}, \quad \beta \to \alpha + 2\gamma_{soft}, \quad \cdots$
- Measured cross-section: $\Gamma^{\lambda}_{\beta\to\alpha} + \int_{\lambda}^{E_d} \frac{d^3k}{(2\pi)^3(2\omega)} \Gamma^{\lambda}_{\beta\to\alpha+\gamma_{soft}(k)} + \cdots$

In the sum, we can take IR cutoff $\lambda \to 0$.

✓ We got the IR finite quantity. No realistic problem if we forget the S-matrix.

But, each S-matrix element vanishes: $\lim_{\lambda\to 0} S^{\lambda}_{\alpha,\beta} \to 0.$

Is the S-matrix meaningless?

Dressed state formalism

[Chung (1965), Kibble (1968), Kulish & Faddeev (1970)]

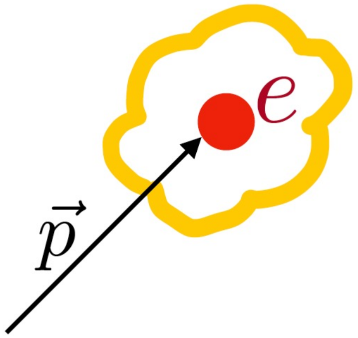
◆ Use of Fock states is probably not appropriate.

- Asymptotic states are usually assumed to be approximated by free states.
- This assumption might be not good for long-range interactions like electromagnetic (and gravitational) force.

● Chung's dressed states

$$||\vec{p}\rangle\rangle = e^{R_C(\vec{p})}\,|\vec{p}\rangle\,, \quad R_C(\vec{p}) = \int \frac{d^3k}{(2\pi)^3(2\omega)}\,\frac{e\,p\cdot\epsilon^A}{p\cdot k}\left(a_A(\vec{k}) - a_A^\dagger(\vec{k})\right)$$

Charged particle has a cloud of coherent photons



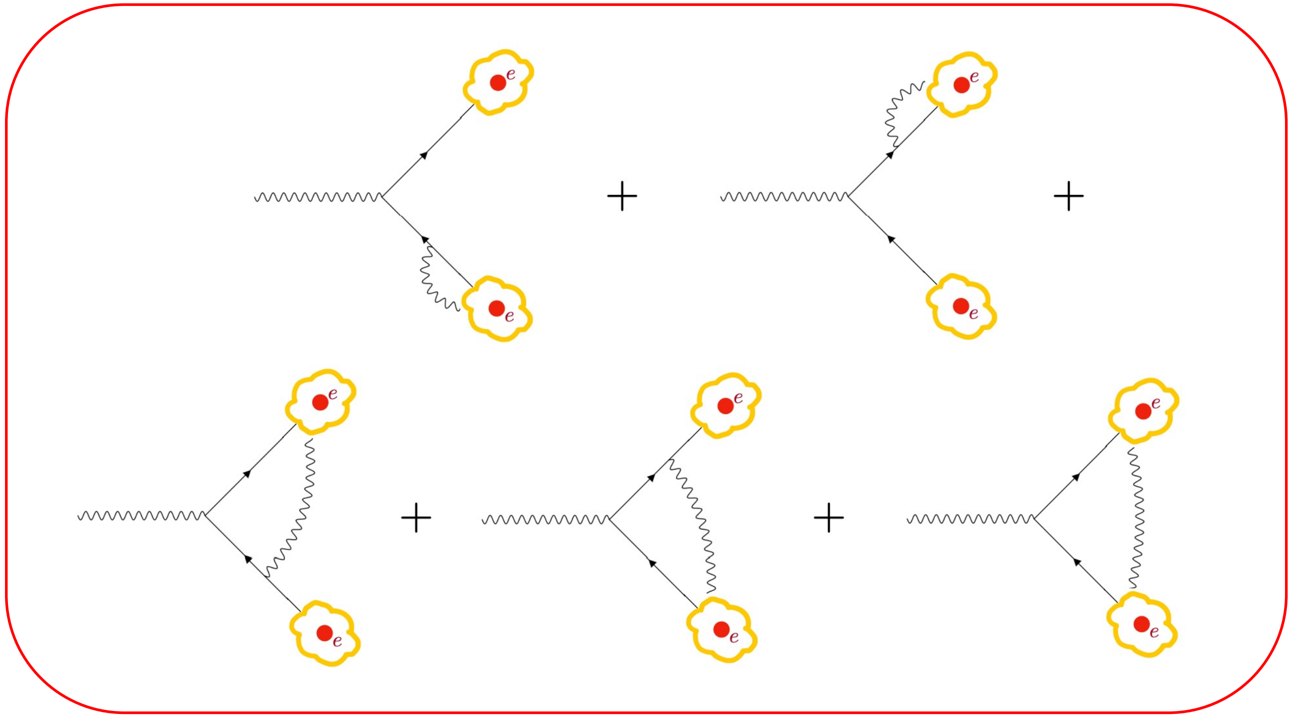
S-matrix for dressed states

[Chung (1965)]

$$ {}_0\langle\alpha|\, e^{-R_C} S e^{R_C}\, |\beta\rangle_0 $$

Cloud of photons also interacts.

New diagrams in addition to usual ones.



These diagrams have IR divs
which cancel those in usual diagrams.

⇒ Totally IR finite!

Fig from Hirai's phD thesis

- S-matrix elements for dressed states are IR finite unlike the Fock states.

A reason of the IR problem

■ Fock space is incomplete

some states orthogonal to all Fock states w/ finite particles

Coherent states

$$|\psi\rangle = e^{R}\,|\psi_0\rangle\,, \qquad R = \int_{\lambda}^{\Lambda} \frac{d^3k}{(2\pi)^3(2\omega)} \frac{e\,p\cdot\epsilon^{A}}{p\cdot k}(a_A(\vec{k}) - a_A^{\dagger}(\vec{k}))$$

Fock

- orthogonal to any finite Fock state in the limit removing the IR cutoff

$$\langle\alpha|\psi\rangle \to 0 \qquad (\lambda \to 0)$$

- time-evolved state $U(t)\,|\beta\rangle$ in QED is roughly this coherent state.

$$\langle\alpha|\,U(t)\,|\beta\rangle \to 0 \qquad (\lambda \to 0)$$

Amplitudes to Fock states vanish.

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Dressed states and asymptotic symmetry

- Dressed states were old subjects. [Chung (1965), Kibble (1968), Kulish & Faddeev (1970)]

- They have been reconsidered in the context of asymptotic sym.

[Mirbabayi & Porrati (2016), Gabai & Sever (2016),
Kapec, Perry, Raclariu & Strominger (2017),
Carney, Chaurette, Neuenfeld & Semenoff (2018),…]

Fock states

- Asymptotic symmetry requires $$\boxed{Q^{\rm hard}_{\rm in}[\epsilon] + Q^{\rm soft}_{\rm in}[\epsilon] = Q^{\rm hard}_{\rm fin}[\epsilon] + Q^{\rm soft}_{\rm fin}[\epsilon]}$$
- However, usual Fock states $Q^{\rm soft}[\epsilon] = 0$
- For non-trivial scattering, general hard charges change $Q^{\rm hard}_{\rm in}[\epsilon] \neq Q^{\rm hard}_{\rm fin}[\epsilon]$

  except for the global part $\epsilon = const.$
- Thus, the scattering between Fock states cannot satisfy the conservation law. Or, the symmetry does not allow such scattering.
- This is consistent with the result $\lim_{\lambda\to 0} S^{\lambda}_{\beta,\alpha} = 0\,.$

  [Kapec, Perry, Raclariu & Strominger (2017)]

IR divergence is necessary

$$\lim_{\lambda \to 0} S^{\lambda}_{\beta,\alpha} = 0$$

- The amplitude for the prohibited process by asymptotic sym vanishes thanks to the IR divergence.

- IR divergence is necessary to maintain asympt sym.
  and the IR problem is not a problem in this sense.

Necessity of dressed states

- Fock states are not enough to maintain the asympt sym because they have vanishing soft charges $Q^{\mathrm{soft}}[\epsilon] = 0$

- Dressed states (coherent state of soft photons) can have non-vanishing soft charges $Q^{\mathrm{soft}}[\epsilon] \neq 0$

- Our questions:
    - What dressed states lead to non-vanishing amplitude $\lim_{\lambda \to 0} S^{\lambda}_{\beta,\alpha} \neq 0$?
    - Is it consistent with the asymptotic symmetry?

IR factorization

[Hirai & SS (2022)]

- General dressed asymptotic states

$$|\alpha\rangle_{\text{in}} = e^{C_\alpha}\,|\alpha\rangle_0\,, \quad |\beta\rangle_{\text{out}} = e^{D_\beta}\,|\beta\rangle_0$$

$$C_\alpha = \int_\lambda^{\Lambda_s} \frac{d^3k}{(2\pi)^3(2\omega)}\left[C_\alpha^A(\vec{k})a_A(\vec{k}) - C_\alpha^{A*}(\vec{k})a_A^\dagger(\vec{k})\right]$$

soft photons

■ We can evaluate IR factor for general dressed states.

$$\Rightarrow \quad \boxed{S_{\beta,\alpha} = \underline{S_{\beta,\alpha}^{\text{soft}}(\Lambda_s,\lambda)}\,S_{\beta,\alpha}^{\text{hard}}(\Lambda_s)}$$

This is generalization of IR factorization formula for Fock states

$$S_{\beta,\alpha} = \left(\frac{\lambda}{\Lambda_s}\right)^{\#} S_{\beta,\alpha}^{\Lambda_s}$$

General IR factor

[Hirai & SS (2022)]

$$S_{\beta,\alpha} = \underline{S^{\rm soft}_{\beta,\alpha}(\Lambda_s,\lambda)}S^{\rm hard}_{\beta,\alpha}(\Lambda_s)$$

$$= e^{N_{D_\beta,C_\alpha}} \quad \text{where} \quad {\rm Re}(N_{D_\beta,C_\alpha}) = -\frac{1}{2}\int_\lambda^{\Lambda_s}\frac{d^3k}{(2\pi)^3(2\omega)}\underline{|R^A_{\beta,\alpha} - C^A_\alpha + D^A_\beta|^2} \leq 0$$

$$R^A_{\beta,\alpha} = R^A_\beta - R^A_\alpha, \qquad R^A_\psi := \sum_{n\in\psi} e_n \frac{p_n\cdot\epsilon^A}{p_n\cdot k} \quad (\psi = \alpha,\beta)$$

The integrand generally has $1/k^2$ terms and thus the integral gives $\log\lambda$ div.

It means the IR problem $\boxed{\lim_{\lambda\to 0} S_{\beta,\alpha} = 0}$

We have to choose the dress so that they cancel the divergences
to obtain non-vanishing S-matrix elements.

Dress code

[Hirai & SS (2022)]

$$\boxed{S_{\beta,\alpha} = e^{N_{D_\beta,C_\alpha}} S^{\rm hard}_{\beta,\alpha}(\Lambda_s)}$$

$$\mathrm{Re}(N_{D_\beta,C_\alpha}) = -\frac{1}{2}\int_\lambda^{\Lambda_s} \frac{d^3k}{(2\pi)^3(2\omega)} \underline{|R^A_{\beta,\alpha} - C^A_\alpha + D^A_\beta|^2} \leq 0$$

- Dress code for non-vanishing S-matrix elements:

$$\boxed{D_\beta - C_\alpha = -R_{\beta,\alpha} + o(k^{-1})}$$

$$\lim_{k\to 0} k\, o(k^{-1}) = 0$$

- A choice is $C^A_\alpha = -\sum_{n\in\alpha} e_n \frac{p_n\cdot\epsilon^A}{p_n\cdot k}$, $D^A_\beta = -\sum_{n\in\beta} e_n \frac{p_n\cdot\epsilon^A}{p_n\cdot k}$ (Chung's dress)

$\Rightarrow$ $S_{\beta,\alpha} = S^{hard}_{\beta,\alpha}(\Lambda_s)$ IR finite!

- Other choices than Chung's are possible.

Quantum memory effect

[Hirai & SS (2022)]

- Dress code

$$D_\beta - C_\alpha = -R_{\beta,\alpha} + o(k^{-1})$$

final initial change of hard info

$$\Rightarrow \quad Q^{\mathrm{soft}}_{\mathrm{fin}}[\epsilon] - Q^{\mathrm{soft}}_{\mathrm{in}}[\epsilon] = -Q^{\mathrm{hard}}_{\mathrm{fin}}[\epsilon] + Q^{\mathrm{hard}}_{\mathrm{in}}[\epsilon]$$

Hard charges are the same as those for classical charged point particles.

- Dress code is the electromagnetic memory effect!

  The result is consistent with the asymptotic symmetry.

- Amplitudes for prohibited transition btw dressed states vanishes thanks to the IR divergence.

Superselection rule

$$|\text{in}\rangle = f_\alpha |\alpha\rangle + f_{\alpha'} |\alpha'\rangle$$

- Two states cannot interfere if they belong to different sectors of the asymptotic charges.

- We can take a genuine superposition only when the two states have the same asymptotic charges.

- Undressed momentum states for charged particles $|\vec{p}\rangle, |\vec{p'}\rangle$ have different asymptotic charges.

- Wave packet is a superposition of momentum eigenstate $\int d\vec{p}\,\varphi(\vec{p})\,|\vec{p}\rangle$

- Need dress to make wave packets. $\int d\vec{p}\,\varphi(\vec{p}) e^{C(\vec{p})}\,|\vec{p}\rangle_0$

Allowed dress

$$\sum_i f_i \, e^{C_i} \, |\alpha_i\rangle$$

- To have the same asympt charges, the dress should be

$$\boxed{C_i = -R_{\alpha_i} + C + o(k^{-1})}$$

Chung's dress   Common to all i

$$R^A_{\alpha_i} = \sum_{n\in\alpha_i} e_n \frac{p_n \cdot \epsilon^A}{p_n \cdot k}$$

Undressed vs Dressed (1)

The standard inclusive formalism cannot compute S-matrix elements
but can obtain the finite total cross-section.

In any realistic setup, we cannot detect soft photons.
So the inclusive computations with conventional Fock states seem to be enough...

Can we distinguish undress and dress when we can observe only inclusive quantities?

Carney, Chaurette, Neuenfeld & Semenoff (2018) claimed that the answer is yes.

Fock states and Chung states give different results for superposed initial states.

Undressed vs Dressed (2)

Consider superposition of different initial states $|\mathrm{in}\rangle = f_\alpha |\alpha\rangle + f_{\alpha'} |\alpha'\rangle$.

Compute inclusive cross section for the hard final states $|\beta\rangle$.

$$\sigma^{\mathrm{hard}}(\mathrm{in} \to \beta) = \sigma(\mathrm{in} \to \beta) + \sigma(\mathrm{in} \to \beta + \gamma) + \sigma(\mathrm{in} \to \beta + 2\gamma) + \cdots$$

In general, we expect that the cross section contains the interference term of the initial phase $f_\alpha f^*_{\alpha'}$.

However, Carney, Chaurette, Neuenfeld & Semenoff show that

- If we use Fock states (undress), no interference term. Decoherence!
- If we use Chung states (dress), we have the interference term.

The decoherence represents just the superselection rule.

Superselection rule

$$|\mathrm{in}\rangle = f_\alpha\,|\alpha\rangle + f_{\alpha'}\,|\alpha'\rangle$$

Undressed states have only hard charges, and they are different for different configs.

$$Q_{\mathrm{as},\alpha} \neq Q_{\mathrm{as},\alpha'}$$

⇒ always decohered

Two states can interfere only when they have the same asymptotic charges. $Q_{\mathrm{as},\alpha} = Q_{\mathrm{as},\alpha'}$

They should have appropriate soft dresses.

Inclusive cross section for general dress [Hirai & SS (2022)]

Consider $|\mathrm{in}\rangle = \sum_\alpha f_\alpha e^{C_\alpha} |\alpha\rangle_0$

$$\Longrightarrow \quad \sigma^{\mathrm{hard}}(\mathrm{in} \to \beta) = \sum_{\alpha,\alpha'} f_\alpha f^*_{\alpha'} \underline{S^{\mathrm{soft}}_{\alpha',\alpha}} \, S^{\mathrm{hard}}_{\beta,\alpha} S^{\mathrm{hard}\dagger}_{\alpha',\beta}$$

Independent of final dress

soft factor for $\alpha \to \alpha'$

- If $Q_{\mathrm{as},\alpha} \neq Q_{\mathrm{as},\alpha'}$ as undressed states, we have $S^{\mathrm{soft}}_{\alpha',\alpha} = \delta_{\alpha',\alpha}$

$$\Longrightarrow \quad \sigma^{\mathrm{hard}}(\mathrm{in} \to \beta) = \sum_\alpha |f_\alpha|^2 \, S^{\mathrm{hard}}_{\beta,\alpha} S^{\mathrm{hard}\dagger}_{\alpha,\beta}$$

Decoherence

- Chung's dress. We have $S^{\mathrm{soft}}_{\alpha',\alpha} = 1.$ No decoherence
- Other appropriate dresses $S^{\mathrm{soft}}_{\alpha',\alpha} \neq 0$. We can have interference terms.

Allowed dress to have all interference terms

- Only when $\alpha,\ \alpha'$ have the same asymptotic charges, $S^{\text{soft}}_{\alpha',\alpha} \neq 0$

- The decoherence is just the superselection rule.

$$\sum_i f_i\, e^{C_i}\, |\alpha_i\rangle$$

- To have the same asympt charges, the dress should be

$$\boxed{C_i = -R_{\alpha_i} + C + o(k^{-1})}$$

Chung's dress　　Common to all i

$$R^A_{\alpha_i} = \sum_{n\in\alpha_i} e_n \frac{p_n \cdot \epsilon^A}{p_n \cdot k}$$

Soft graviton

- Gravity also has the IR problem.
  (soft graviton theorem is similar to soft photon theorem)

- We should use states dressed by soft gravitons to obtain IR-safe S-matrix.

- We expect that there is a decoherence problem similar to QED.

- The decoherence is due to the IR divergence (or superselection rule of gravity), and so we have to be careful when we discuss decoherence in quantum gravity.

Conclusion

Future problems

- Extension to gravity & non-Abelian gauge theories
- Development of formalism using dressed states (e.g., dressed wave-packet)
- Subleading computations (work in progress)

Summary

- **The asympt sym plays an important role in the soft sector of QED.**

- Don't use the Fock states if you want finite S-matrix elements.

- IR problem is not a problem, and IR divergences are necessary for symmetry.

- Transition occurs only when $Q_{\mathrm{as},\alpha} = Q_{\mathrm{as},\beta}$

  Choose your dresses fitting with this conservation law.

THANK YOU