Riemann 面への入門

松尾 信一郎

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なぜ Riemann 面か？

- Riemann 面は $\sqrt{z}$ や $\log z$ などの多価関数の解析接続を扱うために生まれた.
- 二年生の「複素関数論」と三年生の「曲線と曲面」の自然な一歩先にある.
- しかも，Riemann 面の理論は現代数学のあらゆる分野の母胎でもあり規範でもある.
- だから，四年生のための現代数学の序奏としてまことにふさわしい.

Georg Friedrich Bernhard Riemann

Riemann 面の三種の神器

- Riemann-Roch の定理

$$h^0(D) - h^0(K - D) = \deg(D) + 1 - g$$

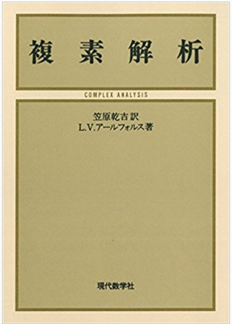
- Serre の双対定理

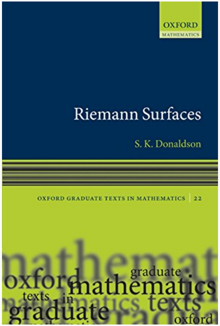
$$h^1(D) = h^0(K - D)$$

- 消滅定理

$$\deg(D) > 2g - 2 \Longrightarrow h^1(D) = 0$$

セミナーの教科書の候補は三冊



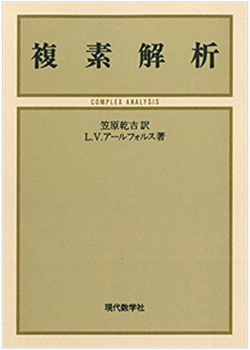


Riemann 面の標準的教科書

小木曽啓示「代数曲線論」

- いわゆる教科書的な教科書.
  代数曲線と Riemann 面は同じ.
- Riemann 面の射影埋込定理を目標とした無駄のない構成.
- 読了すると達成感がある.
- 幾何や代数が好きな人向け.

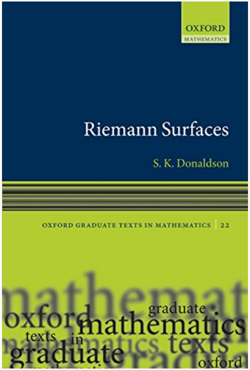
複素関数論から Riemann 面の入口へ



アールフォルス「複素解析」

- 複素関数論の高級な教科書.
- 前半は三年生でやった．後半は，写像定理や楕円関数を経由して，多価関数の解析接続へ.
- 複素関数論の真髄がここに.
- 解析や幾何が好きな人向け.

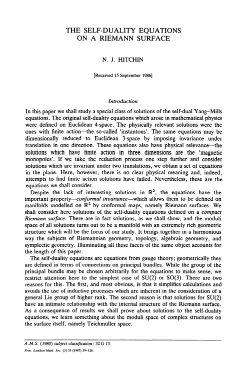
Riemann 面の非標準的教科書



Donaldson, “Riemann Surfaces”, Oxford 2011.

- 現代幾何の観点からの教科書.
- 扱う内容はだいたい普通だが, 説明の仕方が普通ではない.
- 端々の深い含蓄に唸るが, 正直言って, これで初めて Riemann 面を学ぶのは厳しい.
- チャレンジャー向け.

卒業研究の真の目標：Hitchin 方程式



THE SELF-DUALITY EQUATIONS ON A RIEMANN SURFACE

N. J. HITCHIN

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Introduction

In this paper we shall study a special class of solutions of the self-dual Yang–Mills equations. The original self-duality equations which arose in mathematical physics were defined on Euclidean 4-space. The physically relevant solutions were the ones with finite action—the so-called 'instantons'. The same equations may be dimensionally reduced to Euclidean 3-space by imposing invariance under translation in one direction. These equations also have physical relevance—the solutions which have finite action in three dimensions are the 'magnetic monopoles'. If we take the reduction process one step further and consider solutions which are invariant under two translations, we obtain a set of equations in the plane. Here, however, there is no clear physical meaning and, indeed, attempts to find finite action solutions have failed. Nevertheless, these are the equations we shall consider.

Despite the lack of interesting solutions in $\mathbb{R}^2$, the equations have the important property—*conformal invariance*—which allows them to be defined on manifolds modelled on $\mathbb{R}^2$ by conformal maps, namely Riemann surfaces. We shall consider here solutions of the self-duality equations defined on a *compact Riemann surface*. There are in fact solutions, as we shall show, and the moduli space of all solutions turns out to be a manifold with an extremely rich geometric structure which will be the focus of our study. It brings together in a harmonious way the subjects of Riemannian geometry, topology, algebraic geometry, and symplectic geometry. Illuminating all these facets of the same object accounts for the length of this paper.

The self-duality equations are equations from gauge theory; geometrically they are defined in terms of connections on principal bundles. While the group of the principal bundle may be chosen arbitrarily for the equations to make sense, we restrict attention here to the simplest case of SU(2) or SO(3). There are two reasons for this. The first, and most obvious, is that it simplifies calculations and avoids the use of inductive processes which are inherent in the consideration of a general Lie group of higher rank. The second reason is that solutions for SU(2) have an intimate relationship with the internal structure of the Riemann surface. As a consequence of results we shall prove about solutions to the self-duality equations, we learn something about the moduli space of complex structures on the surface itself, namely Teichmüller space.

A.M.S. (1980) subject classification: 32 G 13.

Proc. London Math. Soc. (3) 55 (1987) 59–126.

N. J. Hitchin,

"The self-duality equations on a Riemann surface",

Proc. London Math. Soc. 1987.

$$\begin{cases} d''_A \Phi = 0 \\ F(A) + [\Phi, \Phi^*] = 0 \end{cases}$$

まずは相談に来てください

氏名 松尾 信一郎

居室 理学部 A 館 451 号室

電子メール shinichiroh@math.nagoya-u.ac.jp

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