

Graph Theory

012200083 Kanae Iwata

Exercise 3.4.12 of [GYA]

How many different insertion sequences of the keys 1,4,9,19,41,49 are there that result in a balanced search tree?

Firstly, the definition of a balanced search tree is “for every vertex, the number of vertices in its left and right subtrees differ by at most one.”

Given this definition, the root should be 9 or 19. If this is the case, then the remaining 5 keys can be split evenly in the tree (3:2 or 2:3).

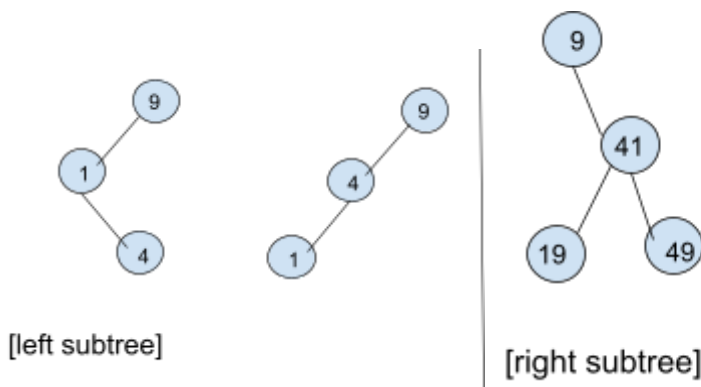
i) root = 9

The keys of the left subtree: 1, 4 $2! = 2$ ways

The keys of the right subtree: 19, 41, 49

one tree, two insertion sequences (41, 19, 49 and 41, 49, 19)

$2 \times 2 = 4$ ways



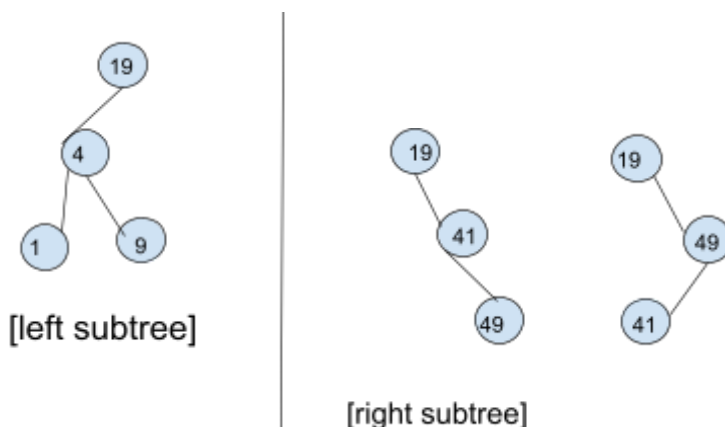
ii) root = 19

The keys of the left subtree: 1, 4, 9

one tree, two insertion sequences (1, 4, 9 and 1, 9, 4)

The keys of the right subtree: 41, 49 $2! = 2$ ways

$2 \times 2 = 4$ ways



$$i) + ii) = 4 + 4 = 8$$

A. 8 ways

Example. [9, 1, 41, 4, 19, 49]

