Order of an Odd Regular Graph

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Claim: For a simple r-regular graph G, if r is odd, the order of G must be even.

Proof: By definition, an r-regular graph must have r edges at each vertex:

$$r|V| = \sum_{v \in V} deg(v) \tag{1}$$

According to handshaking lemma,

$$\sum_{v \in V} deg(v) = 2|E| \tag{2}$$

Hence

$$r|V| = 2|E| \tag{3}$$

On the right hand side we have an even number. On the left hand side, r is odd. For the product of an odd number with another number to be even the other number must be even hence $|V|$ is even ■

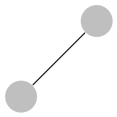
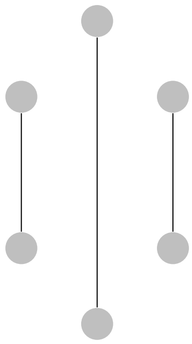
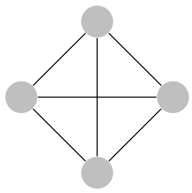
Graph	r (odd)	Order $\|V\|$ (even)	$\sum_{v \in V} deg(v)$
	1	2	1+1 = 2
	1	6	1 + 1 + 1 + 1 + 1 + 1 = 6
	3	4	3 + 3 + 3 + 3 = 12

Table 1: Some examples