Report on Chapter 5. Connectivity

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Definition of Vertex and Edge Connectivity

Let G be a connected graph.

- The vertex connectivity $\kappa_V(G)$ of G is the minimum number of vertices whose removal can either disconnect G or reduce it to a 1-vertex graph.
- The edge connectivity $\kappa_E(G)$ of G is the minimum number of edges whose removal can disconnect G.

Exercise

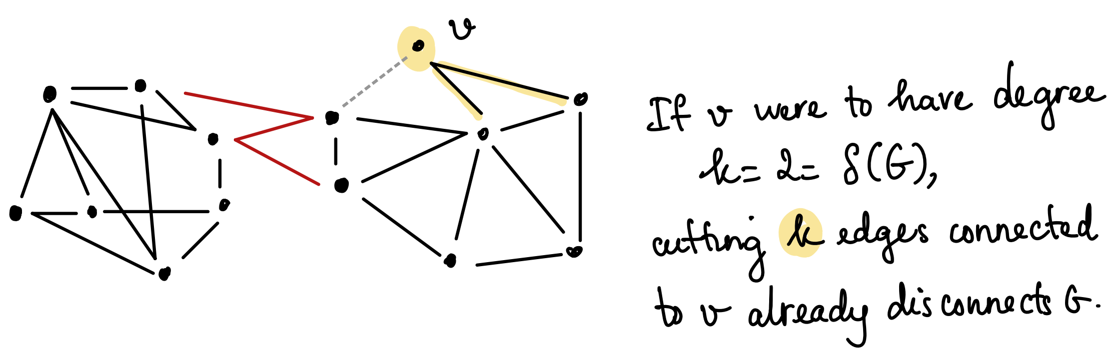
The two connectivities are not independent, one has

$$\kappa_V(G) \leq \kappa_E(G) \leq \delta(G)$$

Proof

1. The edge-connectivity $\kappa_E(G)$ is less than or equal to the minimum degree $\delta(G)$

Let v be a vertex of graph G, with degree $k = \delta(G)$. Suppose that $k = \delta(G) < \kappa_E(G)$. If we remove k edges that are connected to v, we can disconnect G into two subgraphs in which one is a one-vertex graph v. This contradicts the definition of edge connectivity, which is the minimum number of edges required to disconnect G. Therefore, $\kappa_E(G) \leq \delta(G)$.



2. The vertex-connectivity $\kappa_V(G)$ is less or equal than the edge-connectivity $\kappa_E(G)$

Let F be a set of $\kappa_E(G)$ edges such that $G - F$ is disconnected. Such a set exists by definition of F; note that F is a minimal separating set of edges in G. Given F with $|F| = \kappa_E(G)$, there exists at most $|F|$ vertices which disconnect the graph: one for each edge in F (we can choose the endpoint). Thus, $\kappa_V(G) \leq \kappa_E(G)$.