Exercise 6.7: Proof that the Given Graphs are Not Hamiltonian Graphs

THORNTON Alberto-John, TAKEICHI Nozomu

Proof: The Given Graphs are Not Hamiltonian Graphs

Conditions for Hamiltonian cycles

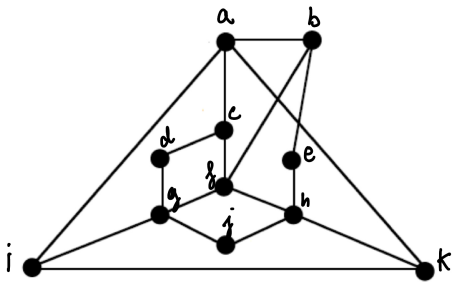
Before starting the proof, let's state the rules for a cycle to be Hamiltonian

i. If a vertex x has degree 2, both incident edges must be used in any Hamiltonian cycle,

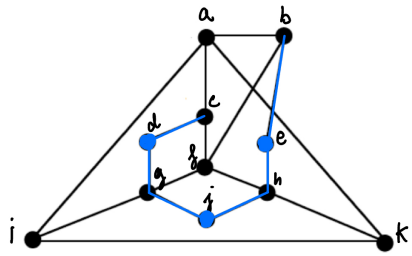
ii. During the construction of a Hamiltonian cycle, no cycle can be formed until all vertices are visited,

iii. If two edges of a given vertex have to be used for a Hamiltonian cycles, then all the other adjacent edges can be disregarded.

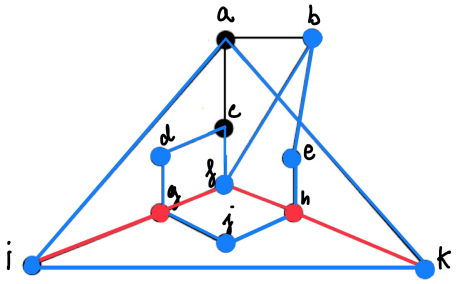
Graph 1



First let's identify vertices with degree 2, these vertices are d, e and g. (Marked in blue). If a vertex has degree 2 we can apply rule i, meaning both incident edges must be used in the Hamiltonian cycle, these edges are: dc, dg, eb, eh, jh and jg. (Marked in blue).

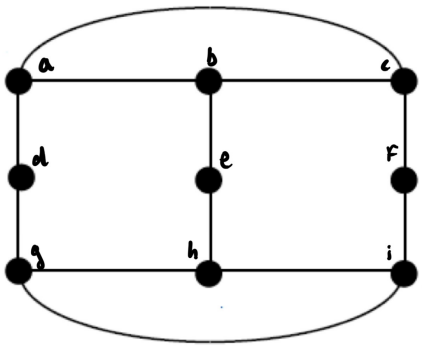


Second, we can apply rule iii to the vertices g and h, edges gi, gf, hf and hk can be disregarded. (Marked in red). Then, three vertices i, k and f become with degree 2 therefore we can apply rule i. (Marked in red).



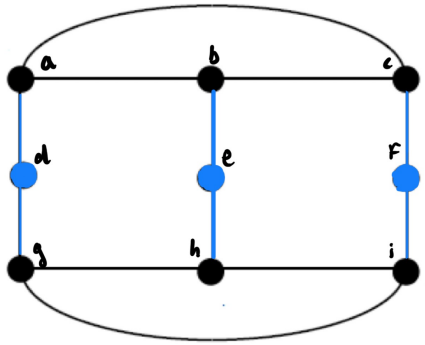
Finally, we can see that this graph has two cycles (marked in blue), which contradicts rule ii. Therefore, this graph is not Hamiltonian.

Graph 2

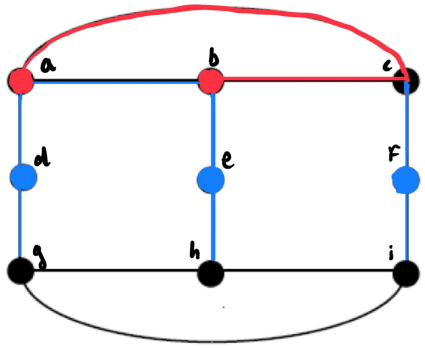


First, let's identify vertices with degree 2, these vertices are d, e and f (marked in blue). If a vertex has degree 2 we can apply rule i, meaning both

incident edges must be used in the Hamiltonian cycle, these edges are: da, dg, eb, eh, fc and di. (Marked in blue).



Second, if the edge ab is on the cycle (marked in blue) and the edge bc is not then rule iii applied to vertex a implies that ac is not in the cycle. (Marked in red)



However, we can then see that edge cf is the only edge in the cycle that is incident to c. Therefore, by rule i no Hamiltonian cycle exists