Reminder XIV

- Basic compartmental models:

S = susceptible, I = infected, R = recovered

population

Either $S+I+R = N$, or $S+I+R = 1$ all variables depend on t

- SI model: $\begin{cases} \dot{s} = \frac{ds}{dt} = -\beta s i \\ \dot{i} = \frac{di}{dt} = \beta s i \end{cases}$ ⤳ logistic equation

SIR model: $\begin{cases} \dot{s} = -\beta s i \\ \dot{i} = \beta s i - \gamma i \\ \dot{r} = \gamma i \end{cases}$ $\beta > 0$ transmission coef, $\gamma > 0$, 1/mean inf. time

no exact solution, but easy simulations.

- Can be generalized in many directions.

- Propagation on graphs, adjacency matrix (a_{ij})

At vertex i: $\begin{cases} \dot{s}_i = -\beta s_i \sum_j a_{ij} i_j \\ \dot{i}_i = \beta s_i \sum_j a_{ij} i_j - \gamma i_i \\ \dot{r}_i = \gamma i_i \end{cases}$ $i = 1, \dots N$ number of vertices

3N coupled equations ... too big.

random variable at vertex i

- Mean field: $[A](t) := \sum_i \mathbb{P}(X_i(t) = A)$ $A = S, I, R$

⤳ $\begin{cases} [\dot{S}](t) = -\beta [SI](t) \\ [\dot{I}](t) = \beta [SI](t) - \gamma [I](t) \\ [\dot{R}](t) = \gamma [I] \end{cases}$ take closure ⇒ small number of equations.

but $[\dot{RI}] = \dots$ new equations