

# Reminder XII

- $G(n, p)$  - model :  $n$  vertices , probability  $p \in [0, 1]$  to have an edge between any 2 vertices.

$$P(|G| = m) = \binom{\binom{n}{2}}{m} p^m (1-p)^{\binom{n}{2} - m} \quad \text{binomial distribution } B(\binom{n}{2}, p)$$

$$E(\deg(x)) =: c = (n-1)p \quad \text{too big!}$$

$$\lim_{\substack{n \rightarrow \infty \\ p = \frac{c}{n-1}}} P(\deg(x) = k) = e^{-c} \frac{c^k}{k!} \quad \text{Poisson distribution } P(c)$$

← we impose it

- Giant component :  $|A^n| \geq \epsilon |G|$  with  $|G| = n \rightarrow \infty$   
↑ connected  
↓ indep. of  $n$   
↑ it exists for  $c > 1$ , with solution  $s = 1 - e^{-cs}$  to belong to it.

- Study of small components (trees, precise distribution)

$$\text{Clustering coefficient } c_x \approx \frac{\# \triangle}{\# \triangle} \quad \text{local or global}$$

$$\text{For } G(n, p), \quad c = c_x = p = \frac{c}{n-1} \quad \text{if we choose this becomes too small}$$

- Weaknesses : 1) Clustering coef. too small

2) degree distribution decreasing too fast

$$\text{Prefer power law : } P(\deg(x) = k) \approx \frac{1}{k^\alpha}, \quad \alpha > 0$$