Lemma 1.3.7 ID 082350784 Yuu HIRAMATSU

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$(T_j)_{j\in\mathbb{N}} \subset \mathcal{D}'(\mathbb{R}^n)$ converges to $T_\infty \in \mathcal{D}'(\mathbb{R}^n)$

$\Leftrightarrow$ $\forall f \in \mathcal{D}(\mathbb{R}^n)$: $\lim_{j\to\infty} T_j(f) = T_\infty(f)$

$$\partial^\alpha T_j(f) = (-1)^{|\alpha|} T_j(\partial^\alpha f)$$

because $\partial^\alpha f \in \mathcal{D}(\mathbb{R}^n)$, so $\lim_{j\to\infty} (-1)^{|\alpha|} T_j(\partial^\alpha f) = (-1)^{|\alpha|} T_\infty(\partial^\alpha f)$

therefore $\lim_{j\to\infty} \partial^\alpha T_j(f) = \partial^\alpha T_\infty(f)$ $\forall f \in \mathcal{D}(\mathbb{R}^n)$

now $(\partial^\alpha T_j)_{j\in\mathbb{N}}$ converges to $\partial^\alpha T_\infty$

when $(T_j)_{j\in\mathbb{N}} \subset \mathcal{D}'(\mathbb{R}^n)$ converges to $T_\infty \in \mathcal{D}'(\mathbb{R}^n)$

$\forall \alpha \in \mathbb{N}^n$