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We define α and β as

「$\limsup_{n\to\infty} a_n = \alpha$, $\liminf_{n\to\infty} a_n = \beta$, $\alpha \geq \beta$」 in sequence $\{a_n\} \subset \mathbb{R}$

① we show 「if $\lim_{n\to\infty} a_n$ exist, α is equal to β」

when α and β are different, we find that no ε (smallar than $\frac{\alpha-\beta}{2}$) satisfies the definition of convergence

definition of convergence is

「for any ε, we can find certain number $n(\varepsilon)$ which satisfies " $m, n \geq n(\varepsilon)$, then $|a_m - a_n| < \varepsilon$」

② we show 「if α is equal to β, $\lim_{n\to\infty} a_n$ exist」

When $\limsup_{n\to\infty} a_n$ is equal to $\liminf_{n\to\infty} a_n$, for any ε, we can find a that satisfies $a'-\varepsilon < a_n < a'+\varepsilon$ (because $\mathbb{R}$ is continuous) exept finite numbers of a_n.

Now, we assume $\{a_n\}$ converges to α, and $a'-\varepsilon < \alpha < a'+\varepsilon$. we define ε' as $\min\{\alpha-(a'-\varepsilon), (a'+\varepsilon)-\alpha\}$

From assumption, For any $n > n_0(\varepsilon')$, $\forall a_n$ satisfies $|a_n - \alpha| < \varepsilon'$.

From ① and ②, we find that

「 $\{a_n\}$ converges $\Leftrightarrow \limsup_{n\to\infty} a_n = \liminf_{n\to\infty} a_n$