Reminder VI

- $f = g$ a.e. if $\{x \mid f(x) \neq g(x)\}$ is of measure 0.
- not necessarily bounded
  If f is Lebesgue measurable and $g = f$ a.e., then g L. m.
- $\{f_j\}$ pointwise bounded $\Rightarrow$ f^*, f_* well defined
  (f^*: lim sup, f_*: lim inf)
  If f_j Lebesgue measurable, then f^*, f_* Lebesgue m.
- Lebesgue measurable partitions $\mathcal{P}$
- For $f \in \mathcal{L}^\infty([a,b])$, $U(f,\mathcal{P})$ and $L(f,\mathcal{P})$
  ($U(f,\mathcal{P})$: upper Lebesgue sum; $L(f,\mathcal{P})$: lower Lebesgue sum)
- $\sup_{\mathcal{P}} L(f,\mathcal{P}) = \inf_{\mathcal{P}} U(f,\mathcal{P}) \Rightarrow f$ Lebesgue integrable.
  $\{$ Riemann int. f. $\} \subset \{$ Lebesgue int. f. $\}$
- For $f \in \mathcal{L}^\infty([a,b])$, f L. integrable $\Leftrightarrow$ f L. measurable.
- If $f = 0$ a.e. then $\int_a^b f(x)\,dx = 0$
  all in $\mathcal{L}^\infty([a,b])$
  $\Rightarrow$ if $f = g$ a.e. then $\int_a^b f(x)\,dx = \int_a^b g(x)\,dx$.
- Extension to $f \notin \mathcal{L}^\infty([a,b])$, consider f_+, f_-
  and $\int_a^b f(x)\,dx = \int_a^b f_+(x)\,dx - \int_a^b f_-(x)\,dx$
  if both exist

$\rightsquigarrow$ $\mathcal{L}([a,b])$ set of Lebesgue integrable functions.