Homework 9

Exercise 1 *Consider the vector field* $f : \mathbb{R}^2 \setminus \{(0,0)\} \ni (x,y) \mapsto \big(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\big) \in \mathbb{R}^2$. *Compute the curve integral for the following curves:*

(i) The curve defined by the circle centered at $(0,0)$ *and of radius* $\sqrt{2}$, *taken in counterclockwise direction, from* $(1,1)$ *to* $(-\sqrt{2},0)$,

(ii) The curve defined by the unit circle centered at $(0,0)$, *taken in counterclockwise direction,*

(iii) The curve defined by the circle centered at $(0,0)$ *and of radius* $r > 0$, *taken in counterclockwise direction.*

Exercise 2 *Consider the vector field* $f : \mathbb{R}^2 \ni (x,y) \mapsto (2x^3y^4 + x, 2x^4y^3 + y) \in \mathbb{R}^2$. *Compute the curve integral along the curve defined by* $c(t) := \big(t\cos(\pi t) - 1, \sin(\pi t/2)\big)$ *for* $t \in [0,1]$.

Exercise 3 *Compute the following integrals:*

$$\iint_\Omega x^2 y \,\mathrm{d}x\,\mathrm{d}y \quad \textit{with} \quad \Omega = [1,2] \times [-3,4],$$
$$\iiint_\Omega \sin(x) yz \,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \quad \textit{with} \quad \Omega = [0,\pi] \times [0,1] \times [0,2].$$

Exercise 4 *1) Compute the integral* $\iint_\Omega (x-y)\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}^2$ *defined by the three lines of equation* $x = 0$, $y = x+2$, *and* $y = -x$,

2) Compute the integral $\iint_\Omega \mathrm{e}^{x+y}\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}^2$ *defined by* $\{(x,y) \in \mathbb{R}^2 \mid |x| + |y| \leq 1\}$,

3) Compute the integral $\iint_\Omega xy\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}_+ \times \mathbb{R}_+$ *defined by the two functions of equation* $y = x^2$ *and* $y = x^4$.