Nagoya University, G30 program Spring 2022

Mathematics Tutorial IIa ❤ ♥ ♡ Instructor : Serge Richard

Homework 8

Exercise 1 *Consider the map*

$$f : \mathbb{R}^2 \ni \ (x, y) \mapsto x^3 - 2xy + 2y^2 - 1 \ \in \mathbb{R}.$$

(i) Show that the implicit function theorem can be applied at the point $(1, 1) \in \mathbb{R}^2$*,*

(ii) Compute the tangent at the point $(1, 1)$ *of the curve of equation* $f(x, y) = 0$*, and determine the position of this curve with respect to the tangent line at this point.*

Exercise 2 *Let* $f : \mathbb{R}^2 \to \mathbb{R}$ *be of class* C^1 *and let* $(x_0, y_0) \in \mathbb{R}^2$ *be a solution of* $f(x_0, y_0) = 0$*. Suppose that* $\partial_y f(x_0, y_0) \neq 0$*. Let* $\phi : (x_0 - \varepsilon, x_0 + \varepsilon) \to \mathbb{R}$ *be the implicit function of class* C^1 *satisfying* $f\big(x, \phi(x)\big) = 0$ *for any* $x \in (x_0 - \varepsilon, x_0 + \varepsilon)$ *and satisfying* $\phi(x_0) = y_0$*. Show that*

$$\phi'(x) = -\frac{\big[\partial_x f\big]\big(x, \phi(x)\big)}{\big[\partial_y f\big]\big(x, \phi(x)\big)}$$

whenever the denominator is not 0

Exercise 3 *Compute the curve integrals in the following situations:*

(i) $f : \mathbb{R}^2 \ni (x, y) \mapsto (x^2 - xy, y^2 - 2xy) \in \mathbb{R}^2$ *and the curve defined by the parabola* $y = x^2$ *from* $(-2, 4)$ *to* $(1, 1)$*,*

(ii) $f : \mathbb{R}^3 \ni (x, y, z) \mapsto (x, z, xz - y) \in \mathbb{R}^3$ *and the curve defined by the segment between* $(0, 0, 0)$ *and* $(1, 2, 4)$*.*

Exercise 4 *a) Consider the vector field* $f : \mathbb{R}^2 \ni (x, y) \mapsto (2xy, x^2 + y^2) \in \mathbb{R}^2$*. Compute the curve integral along the following curves: (i) The segment between* $(0, 0)$ *and* $(1, 1)$*, (ii) The parabola of equation* $y = x^2$ *from the point* $(0, 0)$ *to the point* $(1, 1)$*. What do you observe ?*