Nagoya University, G30 program Spring 2022

Mathematics Tutorial IIa ϟ ☹ ϟ Instructor : Serge Richard

Homework 7

Exercise 1 *a) Let $\Omega \subset \mathbb{R}^2$ be open and let $F : \Omega \to \mathbb{R}^2$ be of class C^1 on Ω. Let us also set $F = (f_1, f_2)$. Show that if*

$$\partial_x f_2 \neq \partial_y f_1$$

then F does not admit a potential function of class C^2.

b) What would be a similar statement for a function $F : \Omega \to \mathbb{R}^3$ if Ω is an open subset of $\mathbb{R}^3$.

c) What about the n-dimensional case, and how many conditions have to be satisfied ?

Exercise 2 *Consider the vector field $F : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}^2$ defined for $(x,y) \neq (0,0)$ by*

$$F(x,y) = \Big(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\Big)$$

(i) Represent graphically this vector field (you can use polar coordinates),

(ii) Can you find a potential function for this vector field, and if so exhibit it.