
Homework 10

Exercise 1 *Compute the integral* $\iiint_\Omega (x+y+z)^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ *with* Ω *the subset of* $\mathbb{R}^3$ *defined by the four planes of equation* $x=0$*,* $y=0$*,* $z=0$*, and* $x+y+z=1$*.*

Exercise 2 *Find the integral of the function* $f:\mathbb{R}^2\to\mathbb{R}$ *defined by* $f(x,y)=\frac{1}{(x^2+y^2+1)^{3/2}}$ *on the disc of radius* R *and centered at the origin of* $\mathbb{R}^2$*.*

Exercise 3 *Find the mass of a spherical ball of radius* R *if the density of the ball at any point is equal to a constant* k *times the distance of that point to the center of the ball.*

Exercise 4 *Find the integral of the function* $f:\mathbb{R}^3\to\mathbb{R}$ *given by* $f(x,y,z)=x^2$ *over the portion of the cylinder defined by* $x^2+y^2=a^2$ *and lying between the planes defined by* $z=0$ *and* $z=b$*, with* $a>0$ *and* $b>0$*.*

Exercise 5 *Compute the area enclosed by the curve given in polar coordinate by* $r^2=\cos(\theta)$*. Sketch this area.*

Exercise 6 *Let* X,Y *be two vectors in* $\mathbb{R}^2$*. Check that the area of the parallelogram spanned by* X *and* Y *is equal to the absolute value of the determinant of the matrix* $(X\ Y)\in M_2(\mathbb{R})$*. More generally, if* $X_1,\dots,X_n$ *are* n *vectors of* $\mathbb{R}^n$*, one writes* $\mathsf{Vol}(X_1,\dots,X_n)$ *for the volume of the* n*-dimensional box spanned by* $X_1,\dots,X_n$*. Why is it natural to have*

$$\mathsf{Vol}(X_1,\dots,X_n)=|\,\mathsf{Det}(X_1\dots X_n)|\ ?$$