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**Homework 1**

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**Exercise 1** Consider a parametric curve in  $\mathbb{R}^2$  given by

$$I \ni t \mapsto (x(t), y(t)) \in \mathbb{R}^2$$

where  $I$  is an interval of  $\mathbb{R}$ , and where  $x : I \rightarrow \mathbb{R}$  and  $y : I \rightarrow \mathbb{R}$  are real functions defined on  $I$ . Represent the following parametric curves:

(i)  $x(t) = \cos(t)$  and  $y(t) = \sin(t)$  for any  $t \in [0, 2\pi]$ ,

(ii)  $x(t) = e^t \cos(t)$  and  $y(t) = e^t \sin(t)$  for any  $t \in \mathbb{R}$ .

**Exercise 2** Write a parametric equation for the tangent line at any point of the curve given by

$$g : \mathbb{R} \ni t \mapsto (e^{3t}, e^{-3t}, t, 1) \in \mathbb{R}^4.$$

**Exercise 3** Consider the parametric curve defined by  $x(t) = te^t$  and  $y(t) = te^{-t}$  for any  $t \in \mathbb{R}$ . Determine the coordinates of the highest point on the curve, and of the leftmost point on the curve.

**Exercise 4** Consider the parametric curve given by

$$c : \mathbb{R} \ni t \mapsto (e^t \cos(t), e^t \sin(t)) \in \mathbb{R}^2$$

Show that the tangent vector to the curve makes a constant angle with the position vector, i.e. with the vector  $c(\cdot)$ .

**Exercise 5** Consider the spiral in  $\mathbb{R}^3$  defined by the function

$$f : \mathbb{R} \ni t \mapsto (\cos(t), \sin(t), t) \in \mathbb{R}^3.$$

Find the length of the spiral between  $t = 0$  and  $t = 1$ .

**Exercise 6** Let  $f : (a, b) \rightarrow \mathbb{R}^d$  be a parametric curve of class  $C^1$ , and let  $\varphi : (c, d) \rightarrow (a, b)$  be of class  $C^1$  and strictly increasing, with  $\varphi(c) = a$  and  $\varphi(d) = b$ . Show that the equality  $L_f = L_{f \circ \varphi}$  holds, or in other words show that the length of the curve does not depend on the parametrization.