SMC Introductn to Probability

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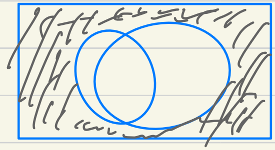
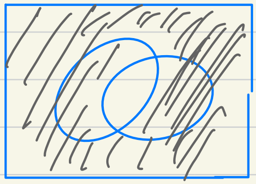
Let $\{A_i : i \in I\}$ be a collection of sets.

Theorem (De Morgan's Laws)

(i) $\left(\bigcup_i A_i\right)^c = \bigcap_i A_i^c$



(ii) $\left(\bigcap_i A_i\right)^c = \bigcup_i A_i^c$



Proof

(i) Consider some $x \in \left(\bigcup_i A_i\right)^c$, $i \in I$

$\iff x \notin \left(\bigcup_i A_i\right)$

$\iff \nexists A_i$ s.t. $x \in A_i$, for $i \in I$

$\iff x \notin A_i$, for $\forall i \in I$

$\iff x \in A_i^c$, $\forall i \in I$

$\iff x \in \bigcap_i A_i^c$ $\angle$

$\therefore$ Since $\forall$ arbitrary $x \in \left(\bigcup_i A_i\right)^c \Rightarrow x \in \bigcap_i A_i^c$
and $\forall$ arbitrary $y \in \bigcap_i A_i^c \Rightarrow y \in \left(\bigcup_i A_i\right)^c$

$\iff \left(\bigcup_i A_i\right)^c = \bigcap_i A_i^c$ by defn of equality of sets.

$\square$

(ii) Consider some $x \in \left(\bigcap_i A_i\right)^c$

$\iff x \notin \left(\bigcap_i A_i\right)$

$\iff \exists j \in I$ s.t. $x \notin A_j$

Define $J := \{ j \mid x \notin A_j \}$

$\iff x \in A_j^c$, $j \in J$

$\iff x \in \left(\bigcup_{j \in J} A_j^c\right) \cup \left(\bigcup_{i \in I/J} \bar{A}_i^c\right)$

$\iff x \in \bigcup_i A_i^c$ h

$\therefore \forall x \in \left(\bigcap_i A_i\right)^c \implies x \in \left(\bigcup_i A_i^c\right)$

and $\forall y \in \left(\bigcup_i A_i^c\right) \implies y \in \left(\bigcap_i A_i\right)^c$

By def^n of equality of sets, $\left(\bigcap_i A_i\right)^c = \left(\bigcup_i A_i^c\right)$

$\square$