

Let X be a random variable that follows Poisson distribution Π_k .

Mean and Variance of Poisson distribution.

$$\Pi_k = \frac{1}{k!} \lambda^k e^{-\lambda} \quad \lambda > 0$$

$$k = 0, 1, 2, \dots$$

• Mean

$$\begin{aligned} E(X) &= \sum_{k=0}^{\infty} k \frac{1}{k!} \lambda^k e^{-\lambda} = \\ &= \lambda \sum_{k=1}^{\infty} \frac{1}{(k-1)!} \lambda^{k-1} e^{-\lambda} \quad k' = k-1 \\ &= e^{-\lambda} \lambda \sum_{k'=0}^{\infty} \frac{1}{k'!} \lambda^{k'} \\ &= \lambda e^{-\lambda} e^{\lambda} \\ &= \lambda \end{aligned}$$

• Variance

$$\begin{aligned} E(X^2) &= \sum_{k=0}^{\infty} k^2 \frac{1}{k!} \lambda^k e^{-\lambda} \\ &= \lambda \sum_{k=1}^{\infty} k \frac{1}{(k-1)!} \lambda^{k-1} e^{-\lambda} \quad k' = k-1 \\ &= \lambda e^{-\lambda} \left[\sum_{k'=0}^{\infty} (k'+1) \frac{1}{k'!} \lambda^{k'} \right] \\ &= \lambda e^{-\lambda} \left[\sum_{k'=0}^{\infty} \frac{1}{k'!} \lambda^{k'} + \sum_{k'=1}^{\infty} \frac{1}{(k'-1)!} \lambda^{k'} \right] \\ &= \lambda e^{-\lambda} \left[e^{\lambda} + \lambda e^{\lambda} \right] \\ &= \lambda(1 + \lambda) \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - E(X)^2 \\ &= \lambda + \lambda^2 - \lambda^2 \\ &= \lambda \end{aligned}$$

$$\Rightarrow \text{Var}(X) = E(X)$$