Homework 5

Exercise 1 *Consider the function* $f : \mathbb{R}^2 \to \mathbb{R}$ *defined by* $f(x,y) = 2x^3 + 6xy - 3y^2 + 2$ *for any* $(x,y) \in \mathbb{R}^2$.

(i) Determine the local extrema of f,

(ii) Does f *possess global extrema ?*

(iii) Consider the segment L *defined by*

$$L = \big\{(x,y) \in \mathbb{R}^2 \mid -2 \leq x \leq 0,\ y = x+1\big\}$$

and determine the global extrema of f *restricted to* L*. Where are these extrema located ?*

Exercise 2 *Consider the function* $f : \mathbb{R}^2 \to \mathbb{R}$ *defined by* $f(x,y) = x\,y\,\mathrm{e}^{-\frac{1}{2}(x^2+y^2)}$.

(i) Study the local extrema of f *(you can use the symmetries of this function),*

(ii) Show that $f(x,y) \to 0$ *as* $\|(x,y)\| \to \infty$,

(iii) Deduce that there exist some global extrema and compute them.

Recall the three norms introduced on $\mathbb{R}^n$:

$$\begin{aligned}\|X\|_2 &:= \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2},\\ \|X\|_1 &:= |x_1| + |x_2| + \cdots + |x_n|,\\ \|X\|_\infty &:= \max_{j\in\{1,\dots,n\}} |x_j|.\end{aligned}$$

Exercise 3 *Show that for any* $X \in \mathbb{R}^n$ *one has*

$$\|X\|_2 \leq \sqrt{n}\|X\|_\infty \leq \sqrt{n}\|X\|_1 \leq n\|X\|_2.$$