

Homework 13

Exercise 1 Let $g : [0, 1] \rightarrow \mathbb{R}_+$ of class C^1 and consider the surface of revolution defined by

$$q : [0, 1] \times [0, 2\pi) \ni (x, \theta) \mapsto \begin{pmatrix} x \\ g(x) \cos(\theta) \\ g(x) \sin(\theta) \end{pmatrix} \in \mathbb{R}^3.$$

Compute the area of this surface.

Exercise 2 Let $\Omega \subset \mathbb{R}^2$ be open and let $g : \Omega \rightarrow \mathbb{R}$ be of class C^1 . We consider the surface of \mathbb{R}^3 parameterized by the function $q : \Omega \rightarrow \mathbb{R}^3$ defined by $q(x, y) = {}^t(x, y, g(x, y))$. Compute the area of the surface $q(\Omega)$. Consider also the surfaces defined by

(i) Ω is the disc of radius 1 centered at $(0, 0) \in \mathbb{R}^2$ and $g(x, y) = x^2 + y^2$,

(ii) Ω is the disc of radius 1 centered at $(0, 0) \in \mathbb{R}^2$ and $g(x, y) = xy$,

Exercise 3 Consider the upper half-sphere S in \mathbb{R}^3 centered in $(0, 0, 0)$ and of radius R , and let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ defined by $f(x, y, z) = x^2 + y^2$. Compute the integral $\iint_S f \, d\sigma$ of f on the upper half-sphere. Same question for f defined by $f(x, y, z) = (x^2 + y^2)z$.

Exercise 4 Consider the vector field f in \mathbb{R}^3 defined by $f(x, y, z) = (x, y, 0)$. Compute the flux of this vector field through the sphere in \mathbb{R}^3 centered at $(0, 0, 0)$ and of radius r .