Homework 1

Exercise 1 *Consider a parametric curve in* $\mathbb{R}^2$ *given by*

$$I \ni t \mapsto \big(x(t), y(t)\big) \in \mathbb{R}^2$$

where I *is an interval of* $\mathbb{R}$, *and where* $x : I \to \mathbb{R}$ *and* $y : I \to \mathbb{R}$ *are real functions defined on* I. *Represent the following parametric curves:*

(i) $x(t) = \cos(t)$ *and* $y(t) = \sin(t)$ *for any* $t \in [0, 2\pi]$,

(ii) $x(t) = \mathrm{e}^t \cos(t)$ *and* $y(t) = \mathrm{e}^t \sin(t)$ *for any* $t \in \mathbb{R}$.

Exercise 2 *Write a parametric equation for the tangent line at any point of the curve given by*

$$g : \mathbb{R} \ni t \mapsto \big(\mathrm{e}^{3t}, \mathrm{e}^{-3t}, t, 1\big) \in \mathbb{R}^4.$$

Exercise 3 *Consider the parametric curve defined by* $x(t) = t\mathrm{e}^t$ *and* $y(t) = t\mathrm{e}^{-t}$ *for any* $t \in \mathbb{R}$. *Determine the coordinates of the highest point on the curve, and of the leftmost point on the curve.*

Exercise 4 *Consider the parametric curve given by*

$$c : \mathbb{R} \ni t \mapsto \big(\mathrm{e}^t \cos(t), \mathrm{e}^t \sin(t)\big) \in \mathbb{R}^2$$

Show that the tangent vector to the curve makes a constant angle with the position vector, *i.e.* with the vector $c(\cdot)$.

Exercise 5 *The curve traced out by a point* P *on the circumference of a circle as the circle rolls along a straight line is called a* cycloid. *Assume that the circle has radius* r *and that the point* P *is initially located at the origin of the* x*-axis.*

(i) Determine the parametric curve defined by the point P,

(ii) Determine the tangent line at any point of the cycloid,

(iii) When is this tangent line horizontal or vertical ?

(iv) Find the area under one arch of the cycloid,

(v) Find the length of one arch of the cycloid.