SEIR MODEL

Correlation models for childhood epidemics

M. J. Keeling, D. A. Rand and A. J. Morris

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PDE

$$\dot{S} = m(E+I+R) - \beta SI,$$
$$\dot{E} = \beta SI - (m+a)E,$$
$$\dot{I} = aE - (m+g)I,$$
$$\dot{R} = gI - mR,$$

total homogeneous mixing. β is often modelled as seasonally varying to mimic the greater amount of contact during the school terms,

$$\beta = 4.93(1 + 0.28\cos(2\pi t))\ \mathrm{day}^{-1},$$

On network with average degree n, and $[X] = \frac{1}{n}\sum_Y [XY],$

$$\left.\begin{aligned}
[\dot{SS}] &= 2m([SE]+[SI]+[SR]) - 2\tau[SSI],\\
[\dot{SE}] &= m([EE]+[EI]+[ER]-[SE])\\
&\quad +\tau([SSI]-[ESI]) - a[SE],\\
[\dot{SI}] &= m([EI]+[II]+[IR]-[SI])\\
&\quad -\tau([ISI]+[SI]) + a[SE] - g[SI],\\
[\dot{SR}] &= m([ER]+[IR]+[RR]-[SR])\\
&\quad -\tau[RSI] + g[SI],\\
[\dot{EE}] &= -2m[EE] + 2\tau[ESI] - 2a[EE],\\
[\dot{EI}] &= -2m[EI] + \tau([ISI]+[SI])\\
&\quad +a([EE]-[EI]) - g[EI],\\
[\dot{ER}] &= -2m[ER] + \tau[RSI] - a[ER] + g[EI],\\
[\dot{II}] &= -2m[II] + 2a[EI] - 2g[II],\\
[\dot{IR}] &= -2m[IR] + a[ER] + g([II]-[IR]).
\end{aligned}\right\}$$

Closure, with clustering coefficient taken into account

$$[XYZ] \approx \frac{n-1}{n}\frac{[XY][YZ]}{[Y]}\left((1-\phi) + \frac{\phi N}{n}\frac{[XZ]}{[X][Z]}\right), \tag{4}$$

With "biologically reasonable assumptions"

(and additional coefficients)

$$\left.\begin{aligned}
\dot{S} &= mN - \tau n\psi - mS,\\
\dot{E} &= \tau n\psi - aE - mE,\\
\dot{I} &= aE - gI - mI,\\
\dot{\psi} &= mI - 2m\psi + aE\frac{S}{N}C_{SE} - g\psi\\
&\quad -\tau[(n-1)\psi + S + \phi(n-1)(C_{II}-1)\psi]\frac{\psi}{S}\\
&\approx mI - 2m\psi + a\frac{(n-1)}{n}EC_{SS}\\
&\quad \times\left[(1-\phi)\frac{S}{N} + \phi\frac{\psi}{I}\right]\\
&\quad -g\psi - \tau[(n-1)\psi + S]\frac{\psi}{S}.
\end{aligned}\right\} \tag{7}$$

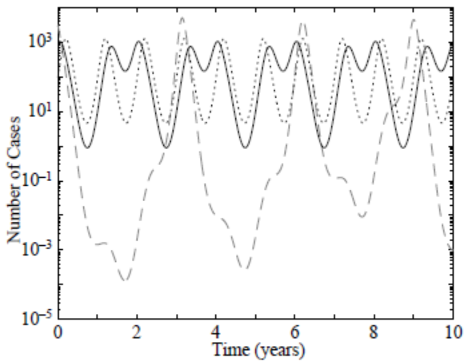


Figure 3. Typical output from the SEIR model in the chaotic regime (grey dashed line) together with regular annual (dotted) and biennial (solid) cycles from the simplified pair model (equation (7)) which do not suffer from the low troughs. The parameters are those given in table 2 and with a population of one million; the annual cycles are produced by $\phi = 0.2$.

Table 1. *Parameter values used in equation (7)*

parameter	value
m^{-1}	50 years
a^{-1}	8 days
g^{-1}	5 days
τ	7.35×10^{-3}
n	$665(1 + 0.28\cos(2\pi t))$
ϕ	0.08
C_{SS}	2.0

as seen in the standard SEIR approach. These models lack the refinements that arise when age structure is included, but demonstrate the strong stabilizing effects that pair correlations confer.

Next: formulation of an age structured pair (ASP) model

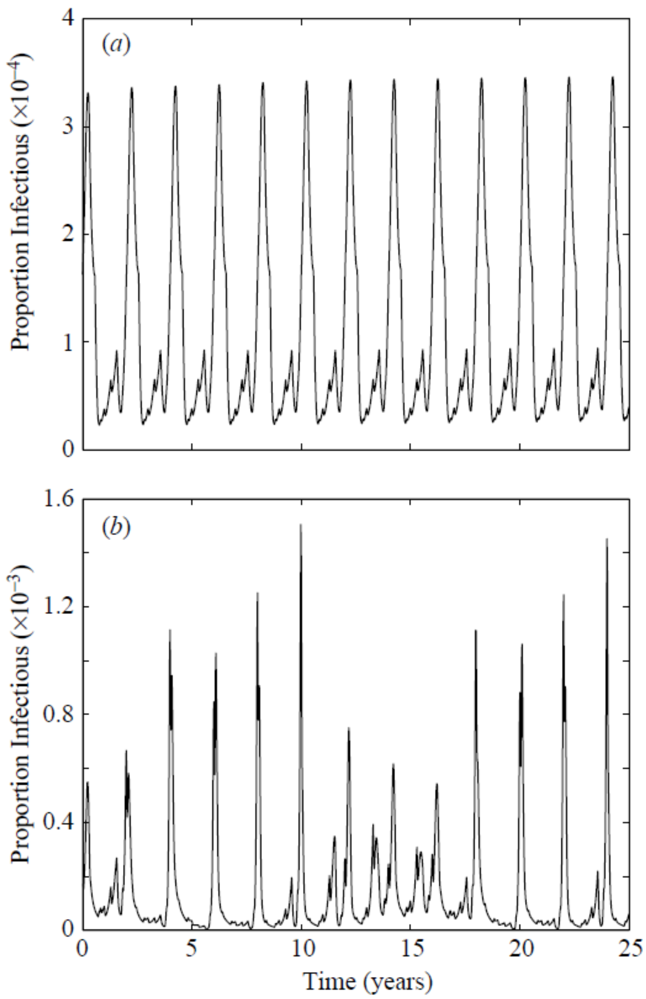
Table 2. *Parameter values used in the ASP equation*

($\mathbb{P}(n, \phi)$ is the probability of having n neighbours with a connectedness of ϕ. For within each age class all transmissibilities are $\tau^{\mathrm{R}} = 0.2$, between age classes all transmissibilities are $\tau^{\mathrm{F}} = 0.5$.)

	connected to			
from	pre-school	primary	secondary	adult
pre-school	$\mathbb{P}(2,1) = 0.2$ $\mathbb{P}(4,0.75) = 0.3$ $\mathbb{P}(6,0.5) = 0.5$	$\mathbb{P}(0,0) = 0.5$ $\mathbb{P}(1,0) = 0.3$ $\mathbb{P}(2,0) = 0.1$ $\mathbb{P}(4,0) = 0.1$	$\mathbb{P}(1,0) = 1.0$	$\mathbb{P}(2,0) = 1.0$
primary school	$\mathbb{P}(0,0) = 0.8$ $\mathbb{P}(2,0) = 0.2$	$\mathbb{P}_{\mathrm{term}}(40,0.85) = 0.5$ $\mathbb{P}_{\mathrm{term}}(40,0.5) = 0.5$ $\mathbb{P}_{\mathrm{hols}}(8,0.6) = 1.0$	$\mathbb{P}(2,0) = 1.0$	$\mathbb{P}(3,0) = 1.0$
secondary school	$\mathbb{P}(1,0) = 1.0$	$\mathbb{P}(2,0) = 1.0$	$\mathbb{P}_{\mathrm{term}}(45,0.78) = 1.0$ $\mathbb{P}_{\mathrm{hols}}(15,0.67) = 1.0$	$\mathbb{P}(5,0) = 1.0$
adult	$\mathbb{P}(2,0) = 1.0$	$\mathbb{P}(2,0) = 1.0$	$\mathbb{P}(2,0) = 1.0$	$\mathbb{P}(25,0.4) = 1.0$

Final pictures

To be compared with



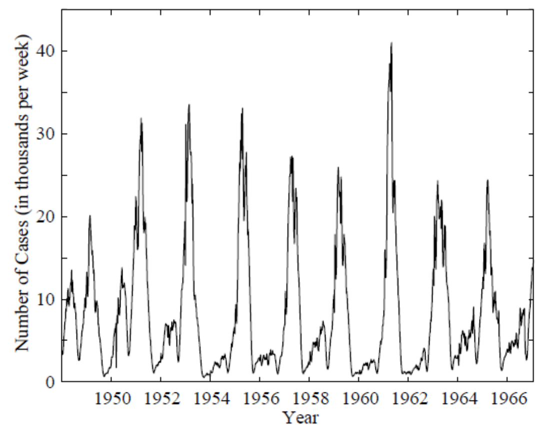


Figure 1. The number of reported cases of measles in England and Wales over the prevaccination period from 1948 to 1966. Strong biennial cycles dominate the dynamics.