Proof of Proposition 2.2.

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Let A_G be a $N \times N$ adjacency matrix. When $r = 1$, $\left(A_G^r\right)_{jk}$ is the number of walks of length 1 from x_j to x_k by the definition of the adjacency matrix. Then, let $r \in N$ be given and suppose $\left(A_G^r\right)_{jk}$ is the number of walks of length r from x_j to x_k. Consider $\left(A_G^{r+1}\right)_{jk}$, and let us compute this entry by using the summation of multiplication of entries of two matrices.

$$\left(A_G^{r+1}\right)_{jk} = \sum_{l=1}^{N} \left(A_G^r\right)_{jl} \left(A_G^1\right)_{lk}. \tag{1}$$

By the assumption and definitions mentioned above, $\left(A_G^r\right)_{jl}$ is the number of walks of length r from x_j to x_l, and $\left(A_G^1\right)_{lk}$ is the number of walks of length 1 from x_l to x_k. Hence, $\left(A_G^r\right)_{jl}\left(A_G^1\right)_{lk}$ is the number of walks of length $r+1$ consisting of two walks, one is the walk of length r from x_j to x_l and another one is the walk of length 1 from x_l to x_k. It follows that $\sum_l \left(A_G^r\right)_{jl}\left(A_G^1\right)_{lk}$ indicates the number of all walks of length $r+1$ from x_j to x_k. Thus, we can regard the l.h.s. of (1) as the number of walks of length $r+1$ from x_j to x_k. One finishes the proof by an induction argument.