1.1.8 GNS rep.

C : C^*-alg φ : positive linear functional $C \to \mathbb{C}$.

$N_\varphi := \{x \in C \mid \varphi(x^* x) = 0\}$.

N_φ is closed left ideal

because φ and mult is cont $\varphi(x_n^* x_n) \to \varphi(x^* x)$,

So we can think quotient of C by N_φ.

$$\begin{array}{ccc} C/N_\varphi \times C/N_\varphi & \to & \mathbb{C} \\ \Psi & & \Psi \\ (x+N_\varphi, y+N_\varphi) & \mapsto & \varphi(y^* x) \end{array}$$

is well defind inner prod on C/N_φ.

$$C/N_\varphi \xrightarrow{\text{completion}} \mathcal{H}_\varphi \text{ : Hilbert sp.}$$

and defin $\pi : C \to B(A/N_\varphi)$.

$$\pi_\varphi(x)(y+N_\varphi) := xy + N_\varphi.$$

This is well-defn and $\|\pi_\varphi(x)\| \leq \|x\|$: bdd.

So we can expand π_φ to $\tilde{\pi}_\varphi$ on $\mathcal{H}_\varphi$.

$\tilde{\pi}_\varphi : C \to B(\mathcal{H}_\varphi)$ is $*$-hom.

we can get GNS rep $(\mathcal{H}_\varphi, \pi_\varphi)$