Motivation

C^* - algebra $\mathcal{C}$

$\begin{cases} K_0(\mathcal{C}) \simeq \{\text{Projections}\}/_{\text{homotopy}} \\ K_1(\mathcal{C}) \simeq \{\text{Unitary el.}\}/_{\text{homotopy}} \end{cases}$

2 Abelian groups, attached to $\mathcal{C}$.

⇝ Classification : not enough information

⇝ Stability : we deal with equivalent classes

If P or U are related to a "physical" system, then all elements of $[P]_0$ or $[U]_1$ share the same property

cyclic cohomology

$[P]_0 \ni p \longmapsto \mathcal{T}(p) \in \mathbb{C}$

$[U]_1 \ni u \longmapsto \mathcal{T}(u) \in \mathbb{C}$

computable on certain elements

Supose $0 \to \mathcal{J} \to \mathcal{E} \to \mathcal{Q} \to 0$

then $\exists$ $\phi : K_1(\mathcal{Q}) \to K_0(\mathcal{J})$ index map

$\psi : K_0(\mathcal{Q}) \to K_1(\mathcal{J})$ exponential map

$\leadsto$ We can link 2 "topological" invariants, one unitary and one projection

→ • bulk-boundary correspondence

• Levinson theorem