**Nagoya University, G30 program** Spring 2020

Mathematics Tutorial IIa ❤ Instructor : Serge Richard

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**Homework 8**

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**Exercise 1** *Consider the map*

$$f:\mathbb{R}^2 \ni \ (x,y)\mapsto x^3-2xy+2y^2-1 \ \in \mathbb{R}.$$

*(i) Show that the implicit function theorem can be applied at the point* $(1,1)\in\mathbb{R}^2$*,*

*(ii) Compute the tangent at the point* $(1,1)$ *of the curve of equation* $f(x,y)=0$*, and determine the position of this curve with respect to the tangent line at this point.*

**Exercise 2** *Consider the map*

$$f:\mathbb{R}^2 \ni \ (x,y)\mapsto \arctan(x+y)+\mathrm{e}^x-2y-1 \ \in \mathbb{R}\ .$$

*(i) Show that the implicit function theorem can be applied at any point* $(x,y)\in\mathbb{R}^2$ *which satisfies* $f(x,y)=0$*,*

*(ii) Let* $\phi$ *be the function which expresses the second coordinates in terms of the first coordinate, and whose existence is justified by the point (i). Compute the Taylor expansion of* $\phi$ *up to the order* 2 *near* $(x,y)=(0,0)$*.*

**Exercise 3** *Let* $f:\mathbb{R}^2\to\mathbb{R}$ *be of class* $C^1$ *and let* $(x_0,y_0)\in\mathbb{R}^2$ *be a solution of* $f(x_0,y_0)=0$*. Suppose that* $\partial_y f(x_0,y_0)\neq 0$*. Let* $\phi:(x_0-\varepsilon,x_0+\varepsilon)\to\mathbb{R}$ *be the implicit function of class* $C^1$ *satisfying* $f\big(x,\phi(x)\big)=0$ *for any* $x\in(x_0-\varepsilon,x_0+\varepsilon)$ *and satisfying* $\phi(x_0)=y_0$*. Show that*

$$\phi'(x)=-\frac{\big[\partial_x f\big]\big(x,\phi(x)\big)}{\big[\partial_y f\big]\big(x,\phi(x)\big)}$$

*whenever the denominator is not* 0

**Exercise 4** *In the setting of the previous exercise and if the function* $f$ *is of class* $C^2$*, compute* $\phi''(x)$ *whenever it is well defined.*