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**Homework 5**

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**Exercise 1** Consider the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  defined by  $f(x, y) = 2x^3 + 6xy - 3y^2 + 2$  for any  $(x, y) \in \mathbb{R}^2$ .

- (i) Determine the local extrema of  $f$ ,
- (ii) Does  $f$  possess global extrema ?
- (iii) Consider the segment  $L$  defined by

$$L = \{(x, y) \in \mathbb{R}^2 \mid -2 \leq x \leq 0, y = x + 1\}$$

and determine the global extrema of  $f$  restricted to  $L$ . Where are these extrema located ?

**Exercise 2** Consider the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  defined by  $f(x, y) = xy e^{-\frac{1}{2}(x^2+y^2)}$ .

- (i) Study the local extrema of  $f$  (you can use the symmetries of this function),
- (ii) Show that  $f(x, y) \rightarrow 0$  as  $\|(x, y)\| \rightarrow \infty$ ,
- (iii) Deduce that there exist some global extrema and compute them.