Nagoya University, G30 program Spring 2020

Mathematics Tutorial IIa Instructor : Serge Richard

Homework 10

Exercise 1 *1) Compute the integral* $\iint_\Omega \mathrm{e}^{x+y}\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}^2$ *defined by* $\{(x,y)\in\mathbb{R}^2 \mid |x|+|y|\leq 1\}$*,*

2) Compute the integral $\iint_\Omega (x-y)\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}^2$ *defined by the three lines of equation* $x=0$*,* $y=x+2$*, and* $y=-x$*,*

3) Compute the integral $\iint_\Omega xy\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}_+\times\mathbb{R}_+$ *defined by the two functions of equation* $y=x^2$ *and* $y=x^4$*.*

Exercise 2 *Compute the integral* $\iiint_\Omega (x+y+z)^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ *with* Ω *the subset of* $\mathbb{R}^3$ *defined by the four planes of equation* $x=0$*,* $y=0$*,* $z=0$*, and* $x+y+z=1$*.*

Exercise 3 *Find the integral of the function* $f:\mathbb{R}^2\to\mathbb{R}$ *defined by* $f(x,y)=\frac{1}{(x^2+y^2+1)^{3/2}}$ *on the disc of radius* R *and centered at the origin of* $\mathbb{R}^2$*.*

Exercise 4 *Find the mass of a spherical ball of radius* R *if the density of the ball at any point is equal to a constant* k *times the distance of that point to the center of the ball.*

Exercise 5 *Find the integral of the function* $f:\mathbb{R}^3\to\mathbb{R}$ *given by* $f(x,y,z)=x^2$ *over the portion of the cylinder defined by* $x^2+y^2=a^2$ *and lying between the planes defined by* $z=0$ *and* $z=b$*, with* $a>0$ *and* $b>0$*.*