Homework 1

Exercise 1 *The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a* cycloid. *Assume that the circle has radius r and that the point P is initially located at the origin of the x-axis.*

(i) Determine the parametric curve defined by the point P,

(ii) Determine the tangent line at any point of the cycloid,

(iii) When is this tangent line horizontal or vertical ?

(iv) Find the area under one arch of the cycloid,

(v) Find the length of one arch of the cycloid.

Exercise 2 *Write a parametric equation for the tangent line at any point of the curve given by*

$$g : \mathbb{R} \ni t \mapsto \big(\mathrm{e}^{3t}, \mathrm{e}^{-3t}, t, 1\big) \in \mathbb{R}^4.$$

Exercise 3 *Consider the spiral in $\mathbb{R}^3$ defined by the function*

$$f : \mathbb{R} \ni t \mapsto \big(\cos(t), \sin(t), t\big) \in \mathbb{R}^3.$$

Find the length of the spiral between $t = 0$ and $t = 1$.

Exercise 4 *Consider the parametric curve given by*

$$c : \mathbb{R} \ni t \mapsto \big(\mathrm{e}^t \cos(t), \mathrm{e}^t \sin(t)\big)$$

Show that the tangent vector to the curve makes a constant angle with the position vector, *i.e.* with the vector $c(\cdot)$.

Exercise 5 *Let $f : (a, b) \to \mathbb{R}^d$ be a parametric curve of class C^1, and let $\varphi : (c, d) \to (a, b)$ be of class C^1 and strictly increasing, with $\varphi(c) = a$ and $\varphi(d) = b$. Show that the equality $L_f = L_{f\circ\varphi}$ holds, or in other words show that the length of the curve does not depend on the parametrization.*