Nguyen Song Ha, Bui Dang Cong

Poisson distribution

I. Summary

In probability theory and statistics, Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant rate and independently of the others. A discrete random variable X is said to have a Poisson distribution with parameter $\lambda \geq 0$ if the probability mass function of X is given by:

$$f(k,\lambda) = P(X = k) = \frac{\lambda^k e^{-\lambda}}{\lambda!} \quad (k \in N)$$

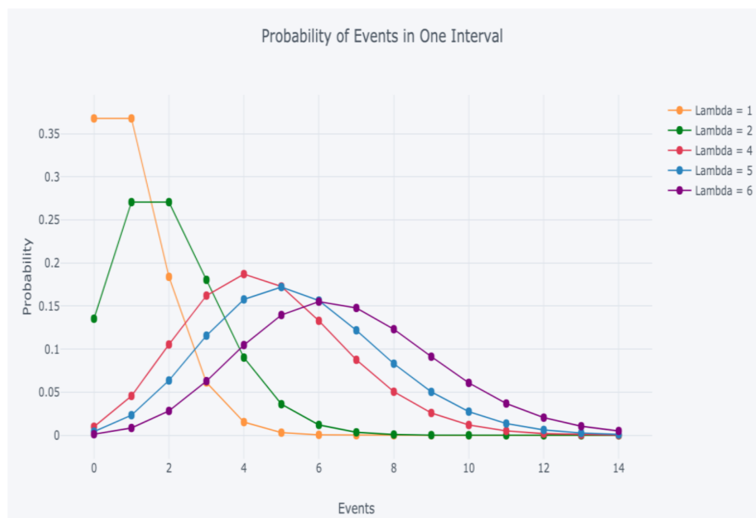
where k is the number of times an event occurs in an interval and k can take values 0, 1, 2,…
By direct calculation, one can get a recurrence relation for the (n+1)$^{\text{th}}$ central moment, given by

$$\mu_{n+1} = \lambda\left(\frac{d\mu_n}{d\lambda} + n\mu_{n-1}\right)$$

with the first two central moments are $\mu_1 = 0$; $\mu_2 = \lambda$.
Poisson distribution is the limiting case of a binomial distribution with a very small probability and large number of trials. In fact, one can prove that the binomial distribution given by

$$P(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

tends to the Poisson distribution with $\lambda = Np$ (the mean number of events after N trials) in the limit when $p \ll 1$ and $n \ll N$.



(image from https://towardsdatascience.com/the-poisson-distribution-and-poisson-process-explained-4e2cb17d459/)
Figure 1: Poisson distribution with different parameter λ

II. Application

Poisson distribution appears naturally in many fields of science such as astronomy, radioactivity process, chemistry, biology and earthquake seismology. In real life, very commonly encountered situations of Poisson distribution are, for example, the number of aircraft/road accidents in any time interval, the number of persons suffering from any rare disease in a large population, the number of births expected during the night in a hospital, etc. Hence, we can use the Poisson distribution to estimate the mean number of events for various purposes, such as preparing for the number of nurses and doctors required during a night shift.

III. References

https://en.wikipedia.org/wiki/Poisson_distribution (online accessed: July 30, 2019)