Exponential Distribution

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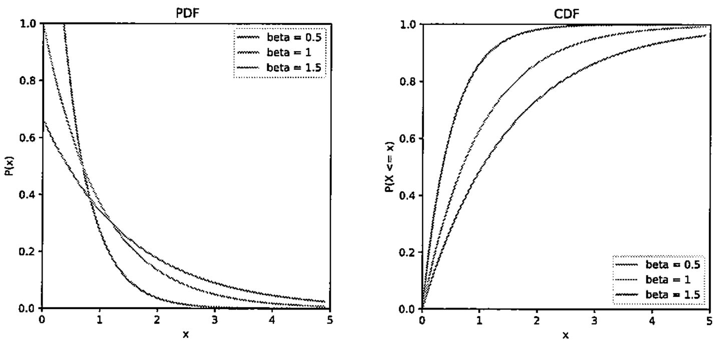
1 General Information

The exponential distribution is a (absolutely) continuous probability distribution.



Parameter	$beta > 0$ (survival parameter)
Support	$0 \leq x < \infty$
Probability density function (pdf)	$f(x \mid \beta) = \frac{1}{\beta} e^{-\frac{x}{\beta}}$
Cumulative distribution function (cdf)	$F(x \mid \beta) = 1 - e^{-\frac{x}{\beta}}$
Moment-generating function (mgf)	$M_x(t) = \frac{1}{1-\beta t}$ (for $t < \frac{1}{\beta}$)
Mean	β
Mode	0
Median	$\beta \ln(2)$
Variance	β^2
Skewness	2
Kurtosis	6

2 Related Distributions

Let $X \sim \text{Exp}(\beta)$ with $\beta > 0$. Then each of the following holds true

$$\begin{aligned} X &\sim \text{Gamma}(1, \tfrac{1}{\beta}), \\ X &\sim \text{Weilbull}(\beta, 1), \\ \sqrt{X} &\sim \text{Rayleigh}(\sqrt{\tfrac{\beta}{2}}), \\ e^{-X} &\sim \text{Beta}(\tfrac{1}{\beta}, 1) \\ ke^{X} &\sim \text{Pareto}(k, \tfrac{1}{\beta}), \\ \alpha - \gamma \log(\tfrac{X}{\beta}) &\sim \text{Gumbel}(\alpha, \gamma). \end{aligned}$$

3 Properties

- closed under scaling by a positive factor, i.e.

$$kX \sim \text{Exp}(\frac{X}{k}) \quad \forall k > 0$$

- memoryless property, i.e.

$$\text{P}(X > s + t \mid X > s) = \text{P}(X > t) \quad \forall s, t \geq 0$$

4 Applications

1. Bolzmann distribution (from physics)

$$p_i \propto e^{-\frac{\varepsilon_i}{\kappa T}}$$

   where ε_i energy, T temperature, and κ bolzmann constant.

   It gives the probability that a system will be in a certain state as a function of that state's energy and the temperature of the system.

2. The exponential distribution describes the time for a continuous process to change state. β is the survival parameter, e.g. time before a machine in a factory breaks.

   Example: On average it takes 10 years for a machine in a certain production line to break, i.e. $\beta = 10$. Let $X \sim \text{Exp}(\beta)$ be the time till the machine breaks. The probability that the machine breaks in the first 5 years is given by

$$\text{P}(X \leq 5) = 1 - e^{-\frac{5}{10}} = 1 - e^{-\frac{1}{2}} \approx 0.3935.$$