Nagoya University, G30 program Spring 2019
Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

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Homework 7

Exercise 1 *Consider the vector field* $F : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}^2$ *defined for* $(x,y) \neq (0,0)$ *by*

$$F(x,y) = \Big(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\Big)$$

(i) Represent graphically this vector field (you can use polar coordinates),

(ii) Can you find a potential function for this vector field, and if so exhibit it.

Exercise 2 *Consider the map*

$$f : \mathbb{R}^2 \ni \ (x,y) \mapsto x^3 - 2xy + 2y^2 - 1 \ \in \mathbb{R}.$$

(i) Show that the implicit function theorem can be applied at the point $(1,1) \in \mathbb{R}^2$*,*

(ii) Compute the tangent at the point $(1,1)$ *of the curve of equation* $f(x,y) = 0$*, and determine the position of this curve with respect to the tangent line at this point.*

Exercise 3 *Consider the map*

$$f : \mathbb{R}^3 \ni \ (x,y,z) \mapsto x^2 - xy^3 - y^2 z + z^3 \ \in \mathbb{R}.$$

(i) Show that the implicit function theorem can be applied at the point $(1,1,1)$*. We shall call* ϕ *the implicit function defined on* $B_\varepsilon\big((1,1)\big)$ *for some* $\varepsilon > 0$ *and which expresses* z *in terms of* x, y *for* z *near the value* 1*,*

(ii) Determine the equation of the plane tangent to the surface defined by $f(x,y,z) = 0$ *at the point* $(1,1,1)$*.*