Nagoya University, G30 program Spring 2019

Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

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Homework 1

Exercise 1 *The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a* cycloid. *Assume that the circle has radius r and that the point P is initially located at the origin of the x-axis.*

(i) Determine the parametric curve defined by the point P,

(ii) Determine the tangent line at any point of the cycloid,

(iii) When is this tangent line horizontal or vertical ?

(iv) Find the area under one arch of the cycloid,

(v) Find the length of one arch of the cycloid.

Exercise 2 *Write a parametric equation for the tangent line at any point of the curve given by*

$$g : \mathbb{R} \ni t \mapsto \big(\mathrm{e}^{3t}, \mathrm{e}^{-3t}, t, 1\big) \in \mathbb{R}^4.$$

Exercise 3 *Consider the spiral in $\mathbb{R}^3$ defined by the function*

$$f : \mathbb{R} \ni t \mapsto \big(\cos(t), \sin(t), t\big) \in \mathbb{R}^3.$$

Determine the equation of the plane perpendicular to the spiral for any $t \in \mathbb{R}$.

Exercise 4 *Find the length of the spiral of the previous exercise between $t = 0$ and $t = 1$.*

Exercise 5 *Consider the parametric curve given by*

$$c : \mathbb{R} \ni t \mapsto \big(\mathrm{e}^t \cos(t), \mathrm{e}^t \sin(t)\big)$$

Show that the tangent vector to the curve makes a constant angle with the position vector, *i.e.* with the vector $c(\cdot)$.

Exercise 6 *Consider a parametric curve $f : \mathbb{R} \to \mathbb{R}^d$ of class C^2, and us call* the osculating plane at *t the plane passing by $f(t)$ and defined by the two vectors $f'(t)$ and $f''(t)$. Obviously this plane is well defined only if these two vectors are not parallel.*

(i) Determine the osculating plane at any t for the spiral defined in Exercise 3.

(i) More generally, for any parametric curve f of class C^2 and for any diffeomorphism φ of class C^2, show that the osculating plane defined by f or by the new parametric curve $f \circ \varphi$ **at the same point** *are equal.*