Nagoya University, G30 program Spring 2018

Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

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Homework 9

Exercise 1 *Consider the map*

$$f:\mathbb{R}^2 \ni (x,y) \mapsto \arctan(x+y) + \mathrm{e}^x - 2y - 1 \in \mathbb{R} .$$

(i) Show that the implicit function theorem can be applied at any point $(x,y) \in \mathbb{R}^2$ which satisfies $f(x,y) = 0$,

(ii) Let ϕ be the function which expresses the second coordinates in terms of the first coordinate, and whose existence is justified by the point (i). Compute the Taylor expansion of ϕ up to the order 2 near $(x,y) = (0,0)$.

Exercise 2 *Let $f:\mathbb{R}^2 \to \mathbb{R}$ be of class C^1 and let $(x_0,y_0) \in \mathbb{R}^2$ be a solution of $f(x_0,y_0) = 0$. Suppose that $\partial_y f(x_0,y_0) \neq 0$. Let $\phi : (x_0-\varepsilon, x_0+\varepsilon) \to \mathbb{R}$ be the implicit function of class C^1 satisfying $f\big(x,\phi(x)\big) = 0$ for any $x \in (x_0-\varepsilon, x_0+\varepsilon)$ and satisfying $\phi(x_0) = y_0$. Show that*

$$\phi'(x) = -\frac{\big[\partial_x f\big]\big(x,\phi(x)\big)}{\big[\partial_y f\big]\big(x,\phi(x)\big)}$$

whenever the denominator is not 0

Exercise 3 *In the setting of the previous exercise and if the function f is of class C^2, compute $\phi''(x)$ whenever it is well defined.*