Nagoya University, G30 program Spring 2018

Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

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Homework 5

Exercise 1 *(i) Compute the Taylor expansion around* $(0,0)$ *and up to the second order of the function*

$$\mathbb{R}^2 \ni (x,y) \mapsto \mathrm{e}^{x^2+xy+y^2} \in \mathbb{R}.$$

(ii) Compute the Taylor expansion around $(0,0)$ *up to the third order of the function*

$$\mathbb{R}^2 \ni (x,y) \mapsto \mathrm{e}^{x+y} \in \mathbb{R}.$$

By fixing then $x = y = 1/2$ *in the polynomial you have obtained, what can you say about the number* e *?*

Exercise 2 *Consider the function* $f : \mathbb{R}^2 \to \mathbb{R}$ *defined by* $f(x,y) = 2x^3 + 6xy - 3y^2 + 2$ *for any* $(x,y) \in \mathbb{R}^2$.

(i) Determine the local extrema of f,

(ii) Does f *possess global extrema ?*

(iii) Consider the segment L *defined by*

$$L = \big\{(x,y) \in \mathbb{R}^2 \mid -2 \leq x \leq 0,\ y = x+1\big\}$$

and determine the global extrema of f *restricted to* L*. Where are these extrema located ?*

Exercise 3 *Consider the function* $f : \mathbb{R}^2 \to \mathbb{R}$ *defined by* $f(x,y) = x\,y\,\mathrm{e}^{-\frac{1}{2}(x^2+y^2)}$.

(i) Study the local extrema of f *(you can use the symmetries of this function),*

(ii) Show that $f(x,y) \to 0$ *as* $\|(x,y)\| \to \infty$,

(iii) Deduce that there exist some global extrema and compute them.

Exercise 4 *Consider the functions* $\varphi : \mathbb{R} \to \mathbb{R}^2$ *defined by* $\varphi(t) := \binom{\cos(t)}{t^2}$*, and* $f : \mathbb{R}^2 \to \mathbb{R}$ *defined by* $f(x,y) := \mathrm{e}^{3x+2y}$*. We consider the composition of these two functions, namely* $F : \mathbb{R} \to \mathbb{R}$ *given by* $F = f \circ \varphi$*. Compute the derivative of this function by two different methods: once by a direct computation, and once as the derivative of a composed function (chain rule).*