
Homework 10

Exercise 1 Compute the curve integrals in the following situations:

- (i) $f : \mathbb{R}^2 \ni (x, y) \mapsto (x^2 - xy, y^2 - 2xy) \in \mathbb{R}^2$ and the curve defined by the parabola $y = x^2$ from $(-2, 4)$ to $(1, 1)$.
- (ii) $f : \mathbb{R}^3 \ni (x, y, z) \mapsto (x, z, xz - y) \in \mathbb{R}^3$ and the curve defined by the segment between $(0, 0, 0)$ and $(1, 2, 4)$,
- (iii) $f : \mathbb{R}^2 \setminus \{(0, 0)\} \ni (x, y) \mapsto \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right)$ and the curve defined by the circle centered at $(0, 0)$ and of radius 2, taken in counterclockwise direction.

Exercise 2 a) Consider the vector field $f : \mathbb{R}^2 \ni (x, y) \mapsto (2xy, x^2 + y^2) \in \mathbb{R}^2$. Compute the curve integral along the following curves: (i) The segment between $(0, 0)$ and $(1, 1)$, (ii) The parabola of equation $y = x^2$ from the point $(0, 0)$ to the point $(1, 1)$. What do you observe ?

Exercise 3 Compute the curve integral

$$\int_C (2x - y)dx + (x + y)dy$$

where C is the circle centered at $(0, 0)$ and of radius R , taken in counterclockwise direction.

Exercise 4 Consider the vector field $f : \mathbb{R}^2 \setminus \{(0, 0)\} \ni (x, y) \mapsto \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right) \in \mathbb{R}^2$. Compute the curve integral for the following curves:

- (i) The curve defined by the circle centered at $(0, 0)$ and of radius $\sqrt{2}$, taken in counterclockwise direction, from $(1, 1)$ to $(-\sqrt{2}, 0)$,
- (ii) The curve defined by the unit circle centered at $(0, 0)$, taken in counterclockwise direction,
- (iii) The curve defined by the circle centered at $(0, 0)$ and of radius $r > 0$, taken in counterclockwise direction.