Def : A tempered distribution is a distribution which is cts on $S'(\mathbb{R}^n)$, i.e. if $f_i \longrightarrow f_\infty$ in $S'(\mathbb{R}^n)$, then $T(f_i) \xrightarrow{i\to\infty} T(f_\infty)$

We set $S'(\mathbb{R}^n)$ for the set of all tempered distribution.

Clearly $S'(\mathbb{R}^n) \subset D'(\mathbb{R}^n)$

Ex : $T_{x \to e^{\|x\|^2}} \notin S'(\mathbb{R}^n)$

Indeed : $T_{x\to e^{\|x\|^2}}(e^{-\|x\|}) \not< \infty$

Prop : The distribution T is tempered if and only if $\exists c > 0$, $m \in \mathbb{N}$ s.t.

$$|T(f)| \leq c \sum_{\substack{\alpha, \beta \in \mathbb{N}^n \\ |\alpha| \leq m \\ |\beta| \leq m}} \sup |x^\beta \partial^\alpha f(x)|. \quad \forall f \in D(\mathbb{R}^n)$$

Def : For any $T \in S'(\mathbb{R}^n)$, we set

$$(\mathcal{F}T)(f) = \langle \mathcal{F}T, f \rangle := \langle T, \mathcal{F}f \rangle \equiv T(\mathcal{F}f) \quad \forall f \in S(\mathbb{R}^n)$$

Since $\mathcal{F}S(\mathbb{R}^n) = S(\mathbb{R}^n)$. $\mathcal{F}T \in S'(\mathbb{R}^n)$

Examples:

1) $\mathcal{F}\delta = T_{\frac{1}{(2\pi)^{n/2}} 1}$

Indeed: $(\mathcal{F}\delta)(f) = \delta(\mathcal{F}f)$

$$= \delta\left(k \to \frac{1}{(2\pi)^{n/2}} \int e^{-ikx} f(x)\,dx\right)$$

$$= \frac{1}{(2\pi)^{n/2}} \int f(x)\,dx = \int \left(\frac{1}{(2\pi)^{n/2}} \cdot 1\right) f(x)\,dx$$

$$= T_{\frac{1}{(2\pi)^{n/2}} 1}(f)$$

2) $\mathcal{F}\, T_1 = (2\pi)^{n/2}\,\delta$

3) $\mathcal{F}\, T_h = T_{\hat{h}}$ if h is nice.