Nagoya University, G30 program Spring 2017

Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

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Homework 9

Exercise 1 *Consider the map*

$$f:\mathbb{R}^2 \ni \ (x,y)\mapsto \arctan(x+y)+\mathrm{e}^x-2y-1 \ \in \mathbb{R}\ .$$

(i) Show that the implicit function theorem can be applied at any point $(x,y)\in\mathbb{R}^2$ which satisfies $f(x,y)=0$,

(ii) Let ϕ be the function which expresses the second coordinates in terms of the first coordinate, and whose existence is justified by the point (i). Compute the Taylor expansion of ϕ up to the order 2 near $(x,y)=(0,0)$.

Exercise 2 *Compute the curve integrals in the following situations:*

(i) $f:\mathbb{R}^2\ni(x,y)\mapsto(x^2-xy,y^2-2xy)\in\mathbb{R}^2$ and the curve defined by the parabola $y=x^2$ from $(-2,4)$ to $(1,1)$.

(ii) $f:\mathbb{R}^3\ni(x,y,z)\mapsto(x,z,xz-y)\in\mathbb{R}^3$ and the curve defined by the segment between $(0,0,0)$ and $(1,2,4)$,

(iii) $f:\mathbb{R}^2\setminus\{(0,0)\}\ni(x,y)\mapsto\big(\frac{x}{\sqrt{x^2+y^2}},\frac{y}{\sqrt{x^2+y^2}}\big)$ and the curve defined by the circle centered at $(0,0)$ and of radius 2, taken in counterclockwise direction.

Exercise 3 *a) Consider the vector field $f:\mathbb{R}^2\ni(x,y)\mapsto(2xy,x^2+y^2)\in\mathbb{R}^2$. Compute the curve integral along the following curves: (i) The segment between $(0,0)$ and $(1,1)$, (ii) The parabola of equation $y=x^2$ from the point $(0,0)$ to the point $(1,1)$. What do you observe ?*

Exercise 4 *Compute the curve integral*

$$\int_C(2x-y)\mathrm{d}x+(x+y)\mathrm{d}y$$

where C is the circle centered at $(0,0)$ and of radius R, taken in counterclockwise direction.

Exercise 5 *Consider the vector field $f:\mathbb{R}^2\setminus\{(0,0)\}\ni(x,y)\mapsto\big(\frac{-y}{x^2+y^2},\frac{x}{x^2+y^2}\big)\in\mathbb{R}^2$. Compute the curve integral for the following curves:*

(i) The curve defined by the circle centered at $(0,0)$ and of radius $\sqrt{2}$, taken in counterclockwise direction, from $(1,1)$ to $(-\sqrt{2},0)$,

(ii) The curve defined by the unit circle centered at $(0,0)$, taken in counterclockwise direction,

(iii) The curve defined by the circle centered at $(0,0)$ and of radius $r>0$, taken in counterclockwise direction.

Exercise 6 *Let $f:\mathbb{R}^n\to\mathbb{R}$ be a continuous function, and let $c:[a,b]\to\mathbb{R}^n$ be a parametric curve of class C^1. What kind of integral can you define with these objects such that the result does not depend on the parametrization of the curve ?*