
Homework 8

Exercise 1 Compute the curve integrals in the following situations:

- (i) $f : \mathbb{R}^2 \ni (x, y) \mapsto (x^2 - 2xy, y^2 - 2xy) \in \mathbb{R}^2$ and the curve defined by the parabola $y = x^2$ from $(-2, 4)$ to $(1, 1)$.
- (ii) $f : \mathbb{R}^3 \ni (x, y, z) \mapsto (x, y, xz - y) \in \mathbb{R}^3$ and the curve defined by the segment between $(0, 0, 0)$ and $(1, 2, 4)$,
- (iii) $f : \mathbb{R}^2 \setminus \{(0, 0)\} \ni (x, y) \mapsto \left(\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}\right)$ and the curve defined by the circle centered at $(0, 0)$ and of radius 2, taken in counterclockwise direction.

Exercise 2 a) Consider the vector field $f : \mathbb{R}^2 \ni (x, y) \mapsto (2xy, x^2 + y^2) \in \mathbb{R}^2$. Compute the curve integral along the following curves:

- (i) The segment between $(0, 0)$ and $(1, 1)$,
- (ii) The parabola of equation $y = x^2$ from the point $(0, 0)$ to the point $(1, 1)$.

What do you observe and can you make a conjecture ?

- b) Can you prove your conjecture ?

Exercise 3 Compute the curve integral

$$\int_C (2x - y)dx + (x + y)dy$$

where C is the circle centered at $(0, 0)$ and of radius R , taken in counterclockwise direction.

Exercise 4 Consider the vector field $f : \mathbb{R}^2 \setminus \{(0, 0)\} \ni (x, y) \mapsto \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right) \in \mathbb{R}^2$. Compute the curve integral for the following curves:

- (i) The curve defined by the circle centered at $(0, 0)$ and of radius $\sqrt{2}$, taken in counterclockwise direction, from $(1, 1)$ to $(-\sqrt{2}, 0)$,
- (ii) The curve defined by the unit circle centered at $(0, 0)$, taken in counterclockwise direction,
- (iii) The curve defined by the circle centered at $(0, 0)$ and of radius $r > 0$, taken in counterclockwise direction.

Exercise 5 Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a continuous function, and let $c : [a, b] \rightarrow \mathbb{R}^n$ be a parametric curve of class C^1 . What kind of integral can you define with these objects such that the result does not depend on the parametrization of the curve ?