Nagoya University, G30 program Spring 2016

Mathematics Tutorial IIa (calculus) Instructor : Serge Richard

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Homework 7

Exercise 1 *Consider the map*

$$f:\mathbb{R}^2 \ni \ (x,y)\mapsto x^3-2xy+2y^2-1 \ \in \mathbb{R}.$$

(i) Show that the implicit function theorem can be applied at the point $(1,1)\in\mathbb{R}^2$*,*

(ii) Compute the tangent at the point $(1,1)$ *of the curve of equation* $f(x,y)=0$*, and determine the position of this curve with respect to the tangent line at this point.*

Exercise 2 *Consider the map*

$$f:\mathbb{R}^2 \ni \ (x,y)\mapsto \arctan(x+y)+\mathrm{e}^x-2y-1 \ \in \mathbb{R}\ .$$

(i) Show that the implicit function theorem can be applied at any point $(x,y)\in\mathbb{R}^2$ *which satisfies* $f(x,y)=0$*,*

(ii) Let ϕ *be the function which expresses the second coordinates in terms of the first coordinate, and whose existence is justified by the point (i). Compute the Taylor expansion of* ϕ *up to the order* 2 *near* $(x,y)=(0,0)$*.*

Exercise 3 *Consider the map*

$$f:\mathbb{R}^3 \ni \ (x,y,z)\mapsto x^2-xy^3-y^2z+z^3 \ \in \mathbb{R}.$$

(i) Show that the implicit function theorem can be applied at the point $(1,1,1)$*. We shall call* ϕ *the implicit function defined on* $B_\varepsilon\big((1,1)\big)$ *for some* $\varepsilon>0$ *and which expresses* z *in terms of* x,y *for* z *near the value* 1*,*

(ii) Determine the equation of the plane tangent to the surface defined by $f(x,y,z)=0$ *at the point* $(1,1,1)$*,*

(iii) Compute the Taylor expansion of ϕ *up to the order* 2 *near the point* $(x,y)=(1,1)$*.*