Nagoya University, G30 program Spring 2015
Linear algebra II Instructor : Serge Richard
Quiz III Name : ____________________

We recall that with any matrix $\mathcal{A} \in M_n(\mathbb{R})$, we associate the linear map $\mathrm{L}_{\mathcal{A}} : \mathbb{R}^n \to \mathbb{R}^n$ defined by $\mathrm{L}_{\mathcal{A}}(X) = \mathcal{A}X$ for any $X \in \mathbb{R}^n$.

Exercise 1 *Circle the correct statement(s) without providing the proof :*

1. *If* 0 *is an eigenvalue of the linear map* $\mathrm{L}_{\mathcal{A}}$, *then* $\mathsf{Det}(\mathcal{A}) = 0$,

2. *If* X *is an eigenvector of* $\mathrm{L}_{\mathcal{A}}$ *for* $\mathcal{A} \in M_n(\mathbb{R})$ *and if* Y *is an eigenvector of* $\mathrm{L}_{\mathcal{B}}$ *for* $\mathcal{B} \in M_n(\mathbb{R})$, *then* $X + Y$ *is an eigenvector of* $\mathrm{L}_{\mathcal{A}+\mathcal{B}}$,

3. *Let* $\mathcal{A} \in M_n(\mathbb{R})$ *and assume that* $\mathrm{L}_{\mathcal{A}}$ *has* n *different eigenvalues, then there must exist a basis of* $\mathbb{R}^n$ *consisting of eigenvectors of* $\mathrm{L}_{\mathcal{A}}$,

4. *If* $\mathcal{A}$ *is an upper triangular matrix, then the eigenvalues of* $\mathrm{L}_{\mathcal{A}}$ *correspond to the diagonal entries of* $\mathcal{A}$,

5. *If* $\mathcal{A} \in M_n(\mathbb{R})$ *is symmetric, then* $\mathrm{L}_{\mathcal{A}}$ *always possesses an eigenvalue different from* 0,

6. *If* $\mathcal{A}, \mathcal{B} \in M_3(\mathbb{R})$ *and if both* $\mathrm{L}_{\mathcal{A}}$ *and* $\mathrm{L}_{\mathcal{B}}$ *have the eigenvalues* 1, 2 *and* 3, *then* $\mathcal{A}$ *and* $\mathcal{B}$ *are similar,*

7. *If* X *is an eigenvector of* $\mathrm{L}_{\mathcal{A}}$, *then* X *is an eigenvector of* $\mathrm{L}_{\mathcal{A}^3}$ *as well,*

8. *If* X *is an eigenvector of* $\mathrm{L}_{\mathcal{A}}$, *then* X *must belong to the kernel of* $\mathrm{L}_{\mathcal{A}}$ *or to the range of* $\mathrm{L}_{\mathcal{A}}$,

9. *If* $\mathsf{Det}(\mathcal{A}) = \mathsf{Det}({}^t\!\mathcal{A})$, *then* $\mathcal{A}$ *must be symmetric,*

10. *If the standard vectors* $E_1, E_2, \dots, E_n$ *are eigenvectors of a linear map* $\mathrm{L}_{\mathcal{A}}$ *in* $\mathbb{R}^n$, *then* $\mathcal{A} \in M_n(\mathbb{R})$ *must be a diagonal matrix.*