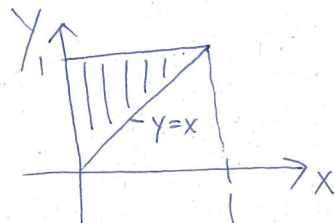


Quiz 4

Name: _____

Explain your solution process clearly.
Write legible.

1. (5 points) Evaluate the double integral $\int_0^1 \int_x^1 (1-y^2)^{-\frac{1}{2}} dy dx$.



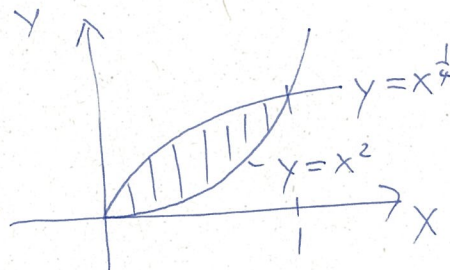
$$\int_0^1 \int_x^1 (1-y^2)^{-\frac{1}{2}} dy dx$$

$$= \int_0^1 \int_0^y (1-y^2)^{-\frac{1}{2}} dy dx = \int_0^1 (1-y^2)^{-\frac{1}{2}} x \Big|_{x=0}^{x=y} dx$$

$$= \int_0^1 (1-y^2)^{-\frac{1}{2}} y dy = -\frac{1}{2} \int_0^1 u^{-\frac{1}{2}} du = -\frac{1}{2} \cdot 2u^{\frac{1}{2}} \Big|_0^1 = 1$$

$u = 1-y^2$
 $du = -2y dy$
 $y=0 \Rightarrow u=1$
 $y=1 \Rightarrow u=0$

2. (5 points) Evaluate the double integral $\iint_D (\sqrt{x} - y^2) dA$ where D is the bounded region enclosed by the curves $y = x^2$ and $y = x^{\frac{1}{4}}$.



$$\iint_D (\sqrt{x} - y^2) dA = \int_0^1 \int_{x^2}^{x^{\frac{1}{4}}} (\sqrt{x} - y^2) dy dx$$

$$= \int_0^1 \left[\sqrt{x} \cdot y - \frac{y^3}{3} \right]_{y=x^2}^{y=x^{\frac{1}{4}}} dx = \int_0^1 \left(\sqrt{x} \cdot x^{\frac{1}{4}} - \frac{x^{\frac{3}{4}}}{3} - \left(\sqrt{x} \cdot x^4 - \frac{x^6}{3} \right) \right) dx$$

$$= \int_0^1 \left(\frac{2}{3} x^{\frac{3}{4}} - \frac{x^{\frac{7}{2}}}{3} + \frac{x^{\frac{5}{2}}}{3} - \frac{x^6}{3} \right) dx$$

$$= \left[\frac{2}{3} \cdot \frac{4}{7} x^{\frac{7}{4}} - \frac{2}{7} x^{\frac{7}{2}} + \frac{1}{21} x^{\frac{7}{2}} - \frac{1}{7} x^7 \right]_0^1 = \frac{8}{21} - \frac{6}{21} + \frac{1}{21} = \frac{1}{7}$$