Math Tutorial II (Calculus)

Quiz 1

Name: ME

Explain your solution process clearly.
Write legible.

1. (10 points) Consider the lines given by the parametric equations $\vec{\ell}_1(t) = t(8,-1,0) + (-1,3,5)$ and $\vec{\ell}_2(t) = t(0,3,1) + (0,3,4)$.

(a) Show that the two lines are not parallel.

The two lines are parallel if $\exists\, c \in \mathbb{R}$ such that: $c(8,-1,0) = (0,3,1)$, so $c \cdot 8 = 0$. Therefore $c = 0$, but $0 \cdot (-1) \neq 3$. Hence there is no $c \in \mathbb{R}$ such that $c(8,-1,0) = (0,3,1)$. Therefore the two lines are not parallel.

(b) Show that the two lines do not intersect.

The two lines intersect if $\exists\, t, s \in \mathbb{R}$ such that

$$\begin{cases} 8t - 1 = 0 & (*) \\ -t + 3 = 3s + 3 & (**) \\ 5 = s + 4 & (***) \end{cases}$$

$\Rightarrow t = \frac{1}{8}$ (by $(*)$)

$s = 1$ (by $(***)$) ; but $(**)$: $-\frac{1}{8} + 3 \neq 3 \cdot 1 + 3$

$\Rightarrow$ The two lines do not intersect.

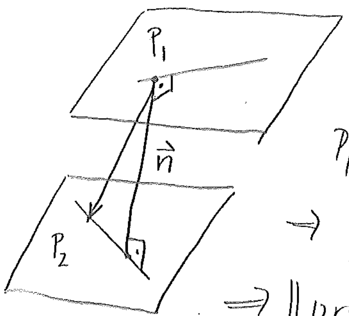
(c) Find a vector perpendicular to both lines.

The vector $\vec{n} = (8,-1,0) \times (0,3,1)$ is perpendicular to both lines. Compute:

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 8 & -1 & 0 \\ 0 & 3 & 1 \end{vmatrix} = \vec{i}(-1) - \vec{j}(8) + 24\vec{k} = (-1,-8,24).$$

$\Rightarrow (-1,-8,24)$ is perpendicular to both lines.

(d) Compute the distance of the two lines.



distance of the lines $= \| \mathrm{proj}_{\vec{n}} \overrightarrow{P_1P_2} \|$ where

$P_1 = (-1,3,5)$, $P_2 = (0,3,4)$ $\Rightarrow \overrightarrow{P_1P_2} = (1,0,-1)$

$\Rightarrow \vec{n} \circ \overrightarrow{P_1P_2} = -1 - 24 = 25$, $\|\vec{n}\|^2 = 1^2 + 64 + 576 = 641$

$\Rightarrow \| \mathrm{proj}_{\vec{n}} \overrightarrow{P_1P_2} \| = \dfrac{25 \cdot \|\vec{n}\|}{\|\vec{n}\|^2} = \dfrac{25}{\sqrt{641}}$