

Problem Set 5 - Math Tutorial Calculus II

1. Consider the function $f(x, y) = 2 + \ln(x^2 + y^2)$.
 - (a) Sketch some level curves and sections of f .
 - (b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.
2. Consider the function $f(x, y) = \cos \sqrt{x^2 + y^2}$.
 - (a) Sketch some level curves and sections of f .
 - (b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.
3. Consider the function $f(x, y) = \frac{1}{x^2 + y^2 + 4}$.
 - (a) Sketch some level curves and sections of f .
 - (b) Use part (a) to give a rough sketch of the graph of $z = f(x, y)$.
4. Is $\{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 4\}$ open, closed or neither?
5. Is $\{(x, y) \in \mathbb{R}^2 : -1 < x < 1\} \cup \{(x, y) \in \mathbb{R}^2 : x = 2\}$ open, closed or neither?
6. Evaluate the following limits, or explain why the limit fails to exist.
 - (a) $\lim_{(x,y,z) \rightarrow (0,0,0)} x^2 + 2xy + yz + z^3 + 2$
 - (b) $\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2+y^2}$
 - (c) $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2+y^2}{x^2+y^2}$
 - (d) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2+2xy+y^2}{x+y}$
 - (e) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4-y^4}{x^2+y^2}$
 - (f) $\lim_{(x,y) \rightarrow (0^+,0^+), x \neq y} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$