

Problem Set 2 - Math Tutorial Calculus II

1. Compute $\vec{a} \circ \vec{b}$, $\|\vec{a}\|$ and $\|\vec{b}\|$ for

(i) $\vec{a} = (4, -1)$ and $\vec{b} = (\frac{1}{2}, 2)$,

(ii) $\vec{a} = \vec{i} + 2\vec{j} - \vec{k}$ and $\vec{b} = 2\vec{i} - 3\vec{j} + 2\vec{k}$.

2. Find the angle between $\vec{a} = \vec{i} + \vec{j} - \vec{k}$ and $\vec{b} = -\vec{i} + 2\vec{j} + 2\vec{k}$.

3. Calculate $proj_{\vec{a}}\vec{b}$ and $proj_{\vec{b}}\vec{a}$ for $\vec{a} = \vec{i} + \vec{j}$ and $\vec{b} = 2\vec{i} + 3\vec{j} - \vec{k}$.

4. Use vectors to show that the diagonals of a parallelogram have the same length if and only if the parallelogram is a rectangle.

5. Evaluate the determinants

(i)

$$\begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix}.$$

(ii)

$$\begin{vmatrix} -2 & 0 & \frac{1}{2} \\ 3 & 6 & -1 \\ 4 & -8 & 2 \end{vmatrix}.$$

6. Calculate $(3\vec{i} - 2\vec{j} + \vec{k}) \times (\vec{i} + \vec{j} + \vec{k})$.

7. Find the area of the triangle having vertices $(1, 1)$, $(-1, 2)$ and $(-2, -1)$.

8. For given vectors \vec{a} , \vec{b} and \vec{c} in \mathbb{R}^3 find expressions for the following vectors:

(a) A vector orthogonal to \vec{a} and \vec{b} .

(b) A vector of length 2 orthogonal to \vec{a} and \vec{b} .

(c) The vector projection of \vec{b} onto \vec{a} .

(d) A vector with the length of \vec{b} and the direction of \vec{a} .

(e) A vector orthogonal to \vec{a} and $\vec{b} \times \vec{c}$.

(f) A vector in the plane determined by \vec{a} , \vec{b} and perpendicular to \vec{c} .