Problem Set 7 - Math Tutorial Calculus II

1. Find an approximate value for $f(0.98, 0.51)$ for $f(x, y) = 3 + \cos(\pi xy)$.
2. Find the gradient of $f(x, y, z) = \frac{x+y}{e^z}$ at $\vec{a} = (3, -1, 0)$.
3. Find $D\mathbf{f}$ for $\mathbf{f}(x, y, z) = (3x - 7y, 5x + 2z, y - 6z, 2x)$ at $\mathbf{a} = (1, 2, 3)$
4. Draw the graphs of the following functions. Find an equation for the line tangent to each graph at the given point $(a, b, f(a, b))$.
   (a) $f(x, y) = 3$, $a = 5$ and $b = 2$,
   (b) $f(x, y) = x^2 + y^2$, $a = -2$, $b = 1$,
   (c) $f(x, y) = \sqrt{x^2 + y^2}$, $a = 2$, $b = 0$,
   (d) $f(x, y) = |x|$, $a = -1$, $b = 3$.

Problem set 7 - solution

#1 see #HW6 - Sol.

#2 $f(x,y,z) = \dfrac{x+y}{e^z}$

$$\Rightarrow Df(x,y,z) = \left(\frac{1}{e^z}, \frac{1}{e^z}, -\frac{x+y}{e^z} \right)$$

$$\Rightarrow (Df)(3,-1,0) = (1,1,-2)$$

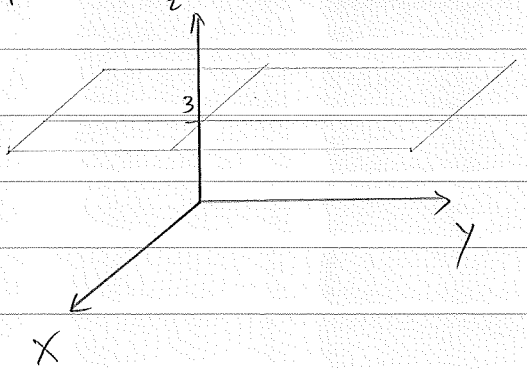
#3

$$\vec{f}(x,y,z) = (3x-7y,\ 5x+2z,\ y-6z,\ 2x)$$

$$\Rightarrow D\vec{f}(x,y,z) = \begin{pmatrix} 3 & -7 & 0 \\ 5 & 0 & 2 \\ 0 & 1 & -6 \\ 2 & 0 & 0 \end{pmatrix} = D\vec{f}(1,2,3)$$

#4 $f(x,y) = 3$ $\Rightarrow f_x(x,y) = 0 = f_y(x,y)$

(a)

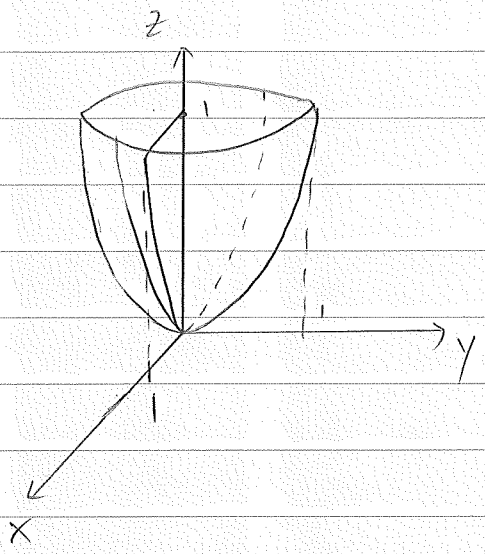


$\Rightarrow$ $z-3=0$ is tangent plane

(b) $f(x,y) = x^2+y^2 \rightarrow f_x(x,y) = 2x \Rightarrow f_x(-2,1) = -4$

$f_y(x,y) = 2y \Rightarrow f_y(-2,1) = 2$

$\Rightarrow 4(x+2) - (y-1) + (z-5) = 0$



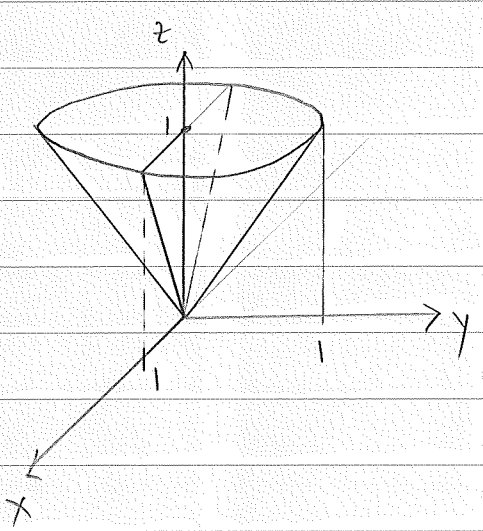
(c) $f(x,y) = \sqrt{x^2+y^2} \Rightarrow f_x(x,y) = \frac{1}{2}(x^2+y^2)^{-\frac{1}{2}} \cdot 2x$

$\Rightarrow f_x(2,0) = \frac{2}{\sqrt{4}} = 1$

$f_y(x,y) = (x^2+y^2)^{-\frac{1}{2}} \cdot y$

$\Rightarrow f_y(2,0) = 0$

$\Rightarrow -(x-2) + (z-2) = 0$

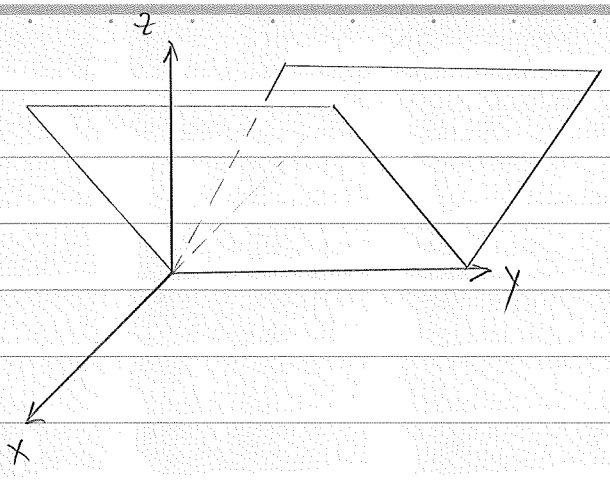


(d) $f(x,y) = |x| \Rightarrow f_x(x,y) = 1$ when $x > 0$

$f_x(x,y) = -1$ when $x < 0$

$f_y(x,y) = 0$

$\Rightarrow f_x(-1,3) = -1$, $f_y(-1,3) = 0 \Rightarrow (x+1) + (z-1) = 0$


z
y
x